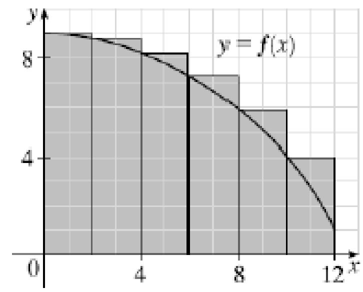
Riemann Review

p. 411: 8-9, 11-12, 15-16, 21, 23, 32-33, 41-42

8. (a) (i) $L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x \quad \left[\Delta x = \frac{12-0}{6} = 2\right]$

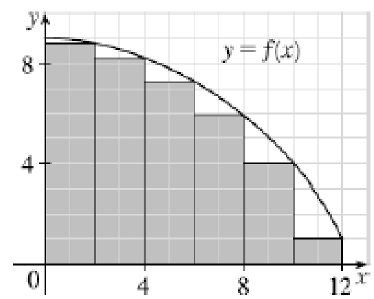
$$= 2[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]$$
$$= 2[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)]$$
$$\approx 2(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1) \ = 2(43.3) = 86.6$$



(ii) $R_6 = L_6 + 2 \cdot (12) - 2 \cdot (0)$

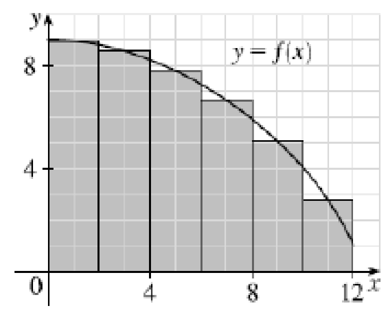
$$\approx 86.6 + 2(1) - 2(9) = 70.6$$

[Add the area of the rightmost lower rectangle and subtract the area of the leftmost upper rectangle.]



(ii) $M_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x$

$$= 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)]$$
$$\approx 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8)$$
$$= 2(39.7) = 79.4$$



(b) Since *f* is *decreasing*, we obtain an *overestimate* by using *left* endpoints; that is, L_6.

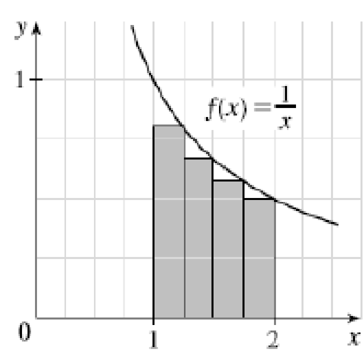
(c) Since *f* is *decreasing*, we obtain an *underestimate* by using *left* endpoints; that is, R_6.

(d) M_6 gives the best estimate because the area of each rectangle appears to be closer to the true area than the overestimates and underestimates in L_6 and R_6.

9. (a) $R_4 = \sum_{i=1}^{4} f(x_i)\Delta x \left[\Delta x = \frac{2-1}{4} = \frac{1}{4}\right] = \left[\sum_{i=1}^{4} f(x_i)\right]\Delta x$

$$= [f(x_1) + f(x_2) + f(x_3) + f(x_4)]\Delta x$$
$$= \left[\frac{1}{5/4} + \frac{1}{6/4} + \frac{1}{7/4} + \frac{1}{8/4}\right]\frac{1}{4} = \left[\frac{4}{5} + \frac{2}{3} + \frac{4}{7} + \frac{1}{2}\right]\frac{1}{4} \approx 0.6345$$

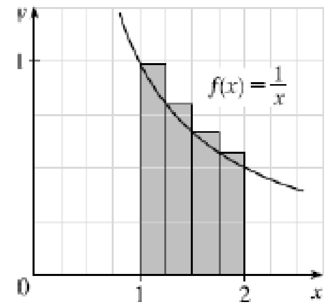
Since *f* is *decreasing* on $[1, 2]$, an *underestimate* is obtained by using the *right* endpoint approximation, R_4.



(b) $L_4 = \sum_{i=1}^{4} f(x_{i-1})\Delta x = \left[\sum_{i=1}^{4} f(x_{i-1})\right]\Delta x$

$= [f(x_0) + f(x_1) + f(x_2) + f(x_3)]\Delta x$

$= \left[\frac{1}{1} + \frac{1}{5/4} + \frac{1}{6/4} + \frac{1}{7/4}\right]\frac{1}{4} = \left[1 + \frac{4}{5} + \frac{2}{3} + \frac{4}{7}\right]\frac{1}{4} \approx 0.7595$



L_4 is an overestimate. Alternatively, we could just add the area of the leftmost upper rectangle and subtract the area of the rightmost lower rectangle, that is $L_4 = R_4 + f(1)\cdot\frac{1}{4} - f(2)\cdot\frac{1}{4}$.

11. (a) $f(x) = 1 + x^2$ and $\Delta x = \frac{2-(-1)}{3} = 1 \Rightarrow$

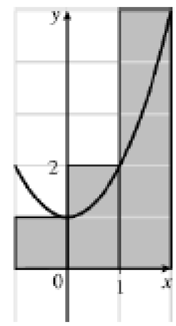
$R_3 = 1\cdot f(0) + 1\cdot f(1) + 1\cdot f(2) = 1\cdot 1 + 1\cdot 2 + 1\cdot 5 = 8.$

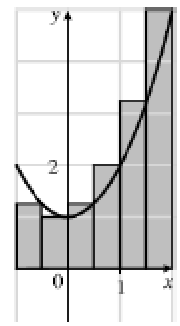
$\Delta x = \frac{2-(-1)}{6} = 0.5 \Rightarrow$

$R_6 = 0.5\cdot[f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2)]$

$= 0.5\cdot(1.25 + 1 + 1.25 + 2 + 3.25 + 5)$

$= 0.5(13.75) = 6.875$



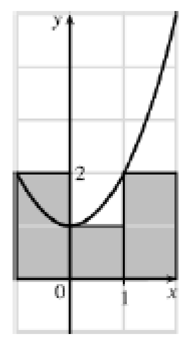


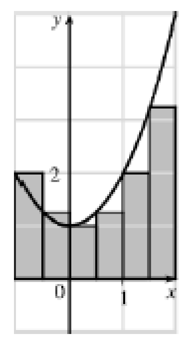
(b) $L_3 = 1\cdot f(-1) + 1\cdot f(0) + 1\cdot f(1) = 1\cdot 2 + 1\cdot 1 + 1\cdot 2 = 5$

$L_6 = 0.5\cdot[f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5)]$

$= 0.5\cdot(2 + 1.25 + 1 + 1.25 + 2 + 3.25)$

$= 0.5(10.75) = 5.375$





(c) $M_3 = 1\cdot f(-0.5) + 1\cdot f(0.5) + 1\cdot f(1.5)$

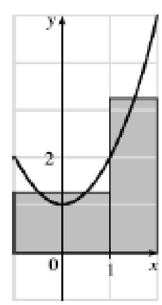
$= 1\cdot 1.25 + 1\cdot 1.25 + 1\cdot 3.25 = 5.75$

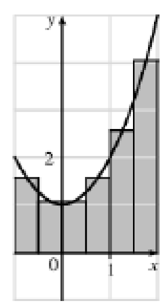
$M_6 = 0.5\cdot[f(-0.75) + f(-0.25) + f(0.25)$

$+ f(0.75) + f(1.25) + f(1.75)]$

$= 0.5\cdot(1.5625 + 1.0625 + 1.0625 + 1.5625 + 2.5625 + 4.0625)$

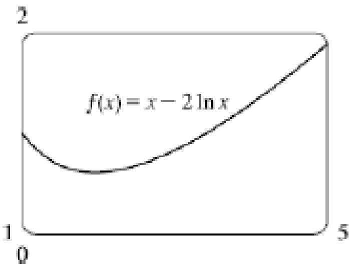
$= 0.5(11.875) = 5.9375$



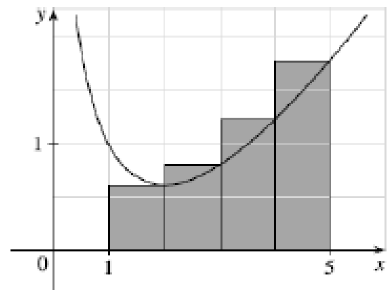


(d) M_6 appears to be the best estimate.

12. (a)



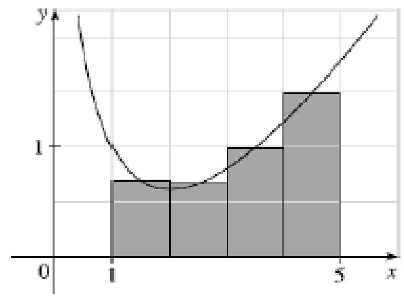
(b) $f(x) = x - 2\ln x$ and $\Delta x = \frac{5-1}{4} = 1 \Rightarrow$



(i) $R_4 = 1 \cdot f(2) + 1 \cdot f(3) + 1 \cdot f(4) + 1 \cdot f(5)$

$= (2 - 2\ln 2) + (3 - 2\ln 3) + (4 - 2\ln 4) + (5 - 2\ln 5)$

≈ 4.425

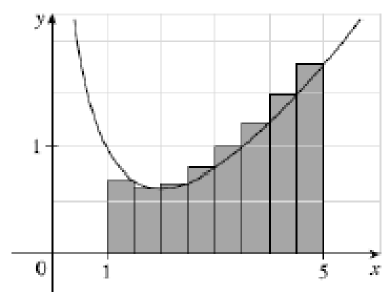


(ii) $M_4 = 1 \cdot f(1.5) + 1 \cdot f(2.5) + 1 \cdot f(3.5) + 1 \cdot f(4.5)$

$= (1.5 - 2\ln 1.5) + (2.5 - 2\ln 2.5)$

$+ (3.5 - 2\ln 3.5) + (4.5 - 2\ln 4.55)$

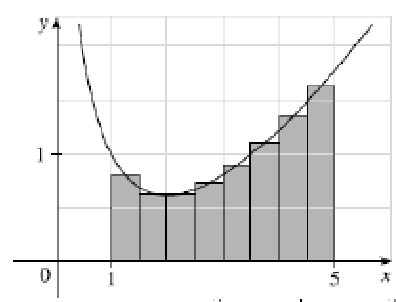
≈ 3.843



(c) (i) $R_8 = \frac{1}{2}[f(1.5) + f(2) + \cdots + f(5)]$

$= \frac{1}{2}[(1.5 - 2\ln 1.5) + (2 - 2\ln 2) + \cdots + (5 - 2\ln 5)]$

≈ 4.134



(ii) $M_8 = \frac{1}{2}[f(1.25) + f(1.75) + \cdots + f(4.75)]$

$= \frac{1}{2}[(1.25 - 2\ln 1.25) + (1.75 - 2\ln 1.75) + \cdots$

$+ (4.75 - 2\ln 4.75)]$

≈ 3.889

15. If f is nonnegative and continuous on $[0, 2]$, then the right Riemann sum R_4 is represented by (**D**) $f(0.5) \cdot \frac{1}{2} + f(2) \cdot \frac{1}{2} + f(1.5) \cdot \frac{1}{2} + f(2) \cdot \frac{1}{2}$

16. If the area under the graph of $y = \sin x$ on the interval $[0, \pi]$ is estimated by L_4, then that estimate is

$L_4 = \left[\sin(0) + \sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{2\pi}{4}\right) + \sin\left(\frac{3\pi}{4}\right)\right]\frac{\pi}{4} = \dfrac{\pi\left(1+\sqrt{2}\right)}{4}$, which is option (**B**).

21. Area of $R \approx T_3 = \frac{1}{2}\big(f(6)+f(12)\big)\cdot 6 + \frac{1}{2}\big(f(12)+f(20)\big)\cdot 8 + \frac{1}{2}\big(f(20)+f(24)\big)\cdot 4$
$=3(5+7)+4(7+4)+2(4+7) = 3(12)+4(11)+2(11) = 36+44+22 = 102$

23. We are given subintervals so we will have 3 rectangles of width 12, and 3 midpoints. Then $M_3 = 12\cdot[f(8)+f(20)+f(32)] = 12\cdot(9+20+10) = 12\cdot 39 = 468$, option (**B**).

32. (**B**)

33. If f is decreasing, continuous, positive and concave down on $[a,b]$, then (**B**) $L_n \geq M_n \geq R_n \geq R_n$.

41. If $f(x)=x^2$ on $[6,4]$ using a regular partition with 100 subintervals, then $\Delta x = \dfrac{6-4}{100} = \dfrac{2}{100} = \dfrac{1}{50.}$ Then

$R_{100} = L_{100} - f(x_0)\cdot\Delta x + f(x_n)\cdot\Delta x = L_{100} - 4^2\cdot\frac{1}{50} + 6^2\cdot\frac{1}{50} = 0.4.$

42. The trapezoidal sum associated with the given function and table is

$61 = \frac{1}{2}\big(3[f(2)+f(5)]+3[f(5)+f(8)]+2[f(8)+f(10)]\big) = \frac{1}{2}\big(3[7+9]+3[9+k]+2[k+11]\big)$

$= \frac{1}{2}(48+27+22+5k) = 97+5k.$ So $122 = 97+5k \Rightarrow 25 = 5k \Rightarrow k = 5.$ This is option (**B**).