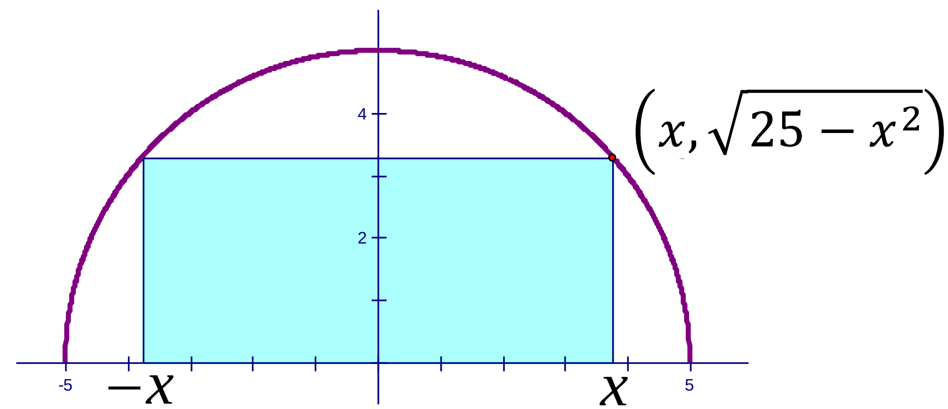
Warm up Problem

The points $(x, 0)$, $(-x, 0)$, $\left(x, \sqrt{25 - x^2}\right)$, and $\left(-x, \sqrt{25 - x^2}\right)$ are the vertices of a rectangle, for $x \leq 5$.

For what value of *x* is the rectangle's area maximum?



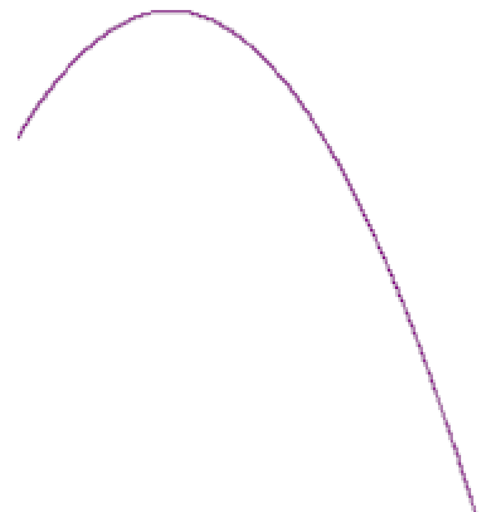
Miscellaneous Theorems

Thm. Extreme Value Theorem

Let $f(x)$ be continuous on the interval $[a, b]$. There exists a point c on $[a, b]$ such that $f(c) \geq f(x)$ for all x on the interval.

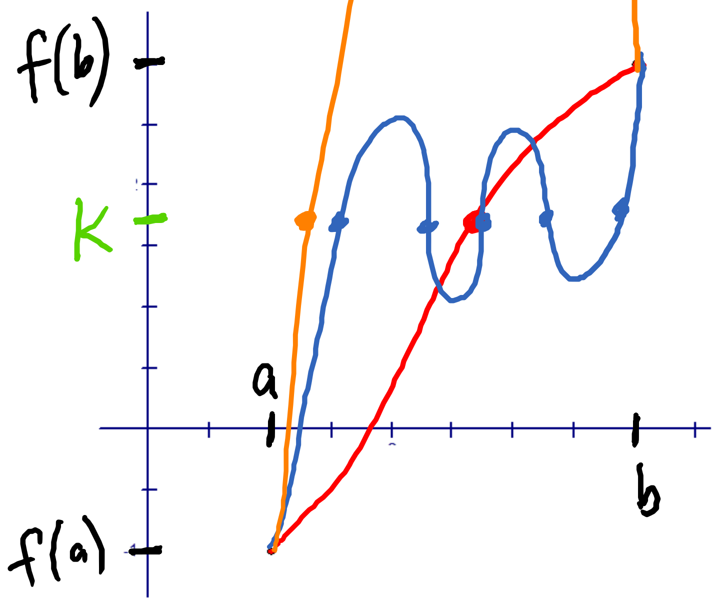
Let $f(x)$ be continuous on the interval $[a, b]$. There exists a point c on $[a, b]$ such that $f(c) \leq f(x)$ for all x on the interval.

There is an absolute max. and an absolute min. on the interval.



Thm. Intermediate Value Theorem

Let $f(x)$ be continuous on the interval $[a, b]$. If k is any number between $f(a)$ and $f(b)$, then there is a point c on $[a, b]$ such that $f(c) = k$.



Every y-coordinate between the endpoints is included

Ex. Show that $f(x) = x^5 - 3x^2 + 1$ has ~~a zero~~ a point such that $f(x) = 10$ on the interval $[-1,2]$.

$f(-1) = -1 - 3 + 1 = -3$

$f(2) = 32 - 12 + 1 = 21$

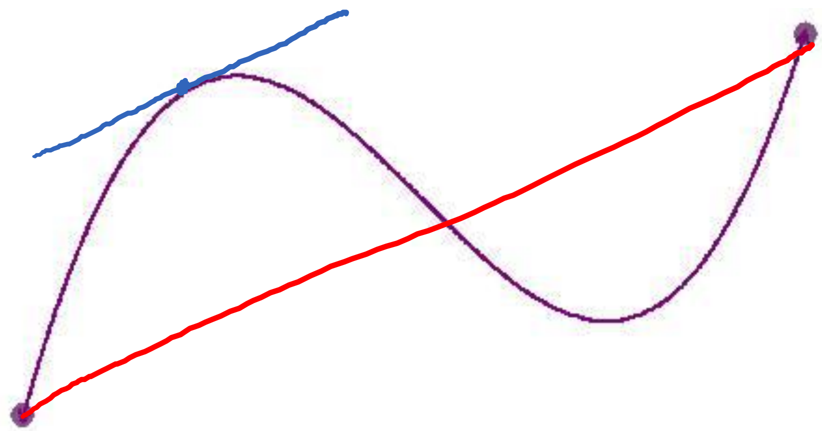
$f(-1) < 10$ and $f(2) > 10$

$\therefore f(x) = 10$ by IVT because $f(x)$ is cont.

Thm. Mean Value Theorem

If $f(x)$ is continuous on the interval $[a, b]$ and differentiable on the interval (a, b), then there is some point c on the interval such that

slope of tangent at a point → $$f'(c) = \frac{f(b) - f(a)}{b - a}$$ ← slope of secant connecting endpoints



Ex. Let $f(x) = x^2 + 2x - 1$. Find c on the interval $[-1,2]$ that satisfies MVT.

$$f'(c) = \frac{f(2) - f(-1)}{2 - (-1)}$$

$$f(2) = 4 + 4 - 1 = 7$$

$$f(-1) = 1 - 2 - 1 = -2$$

$$f'(x) = 2x + 2$$

$$2c + 2 = \frac{7 - (-2)}{3}$$

$$2c + 2 = 3$$

$$2c = 1$$

$$\boxed{c = \frac{1}{2}}$$

Summary

Given two points on a graph of f:

There is one value that f' must attain

- Slope between the endpoints
- Guaranteed by MVT → f must be cont. and diff.

There are many values that f must attain

- All the y's between the endpoints
- Guaranteed by IVT → f must be continuous

Ex. Let f be a twice-differentiable function such that $f(0) = -13$ and $f(7) = 15$. Must there exist a value of c, for $0 < c < 7$, such that $f(c) = 0$? Justify your answer.

IVT

f twice diff. implies f is cont.

$f(0) < 0$ and $f(7) > 0$, therefore $f(c) = 0$ on interval by IVT.

- You need to state that f is continuous to use IVT
- You need to state how you know that f is continuous
  - "Differentiability implies continuity"
- You need to state that the desired value is between the two given values

Theorems Circuit

Beginning in the first cell marked #1, find the requested information. To advance in the circuit, hunt for your answer and mark that cell #2. Continue working in this manner until you complete the circuit.

Unit 4 Progress Check: MCQ

- Do #11-13, 16-18

Unit 5 Progress Check: MCQ Part A

- Do them all

Unit 5 Progress Check: MCQ Part B

- Do them all

Unit 5 Progress Check: MCQ Part C

- Do #1-5