1. Letting $y = ux$ we have

$$(x - ux)\,dx + x(u\,dx + x\,du) = 0$$
$$dx + x\,du = 0$$
$$\frac{dx}{x} + du = 0$$
$$\ln|x| + u = c$$
$$x\ln|x| + y = cx.$$

3. Letting $x = vy$ we have

$$vy(v\,dy + y\,dv) + (y - 2vy)\,dy = 0$$
$$vy^2\,dv + y\left(v^2 - 2v + 1\right)dy = 0$$
$$\frac{v\,dv}{(v-1)^2} + \frac{dy}{y} = 0$$
$$\ln|v-1| - \frac{1}{v-1} + \ln|y| = c$$
$$\ln\left|\frac{x}{y} - 1\right| - \frac{1}{x/y - 1} + \ln y = c$$
$$(x-y)\ln|x-y| - y = c(x-y).$$

5. Letting $y = ux$ we have

$$\left(u^2x^2 + ux^2\right)dx - x^2(u\,dx + x\,du) = 0$$
$$u^2\,dx - x\,du = 0$$
$$\frac{dx}{x} - \frac{du}{u^2} = 0$$
$$\ln|x| + \frac{1}{u} = c$$
$$\ln|x| + \frac{x}{y} = c$$
$$y\ln|x| + x = cy.$$

7. Letting $y = ux$ we have

$$\begin{aligned}(ux - x)\,dx - (ux + x)(u\,dx + x\,du) &= 0\\ \left(u^2 + 1\right)dx + x(u + 1)\,du &= 0\\ \frac{dx}{x} + \frac{u+1}{u^2+1}\,du &= 0\\ \ln|x| + \frac{1}{2}\ln\left(u^2 + 1\right) + \tan^{-1} u &= c\\ \ln x^2\left(\frac{y^2}{x^2} + 1\right) + 2\tan^{-1}\frac{y}{x} &= c_1\\ \ln\left(x^2 + y^2\right) + 2\tan^{-1}\frac{y}{x} &= c_1.\end{aligned}$$

9. Letting $y = ux$ we have

$$\begin{aligned}-ux\,dx + (x + \sqrt{u}\,x)(u\,dx + x\,du) &= 0\\ (x^2 + x^2\sqrt{u}\,)\,du + xu^{3/2}\,dx &= 0\\ \left(u^{-3/2} + \frac{1}{u}\right)du + \frac{dx}{x} &= 0\\ -2u^{-1/2} + \ln|u| + \ln|x| &= c\\ \ln|y/x| + \ln|x| &= 2\sqrt{x/y} + c\\ y(\ln|y| - c)^2 &= 4x.\end{aligned}$$

11. Letting $y = ux$ we have

$$\begin{aligned}\left(x^3 - u^3x^3\right)dx + u^2x^3(u\,dx + x\,du) &= 0\\ dx + u^2x\,du &= 0\\ \frac{dx}{x} + u^2\,du &= 0\\ \ln|x| + \frac{1}{3}u^3 &= c\\ 3x^3\ln|x| + y^3 &= c_1x^3.\end{aligned}$$

Using $y(1) = 2$ we find $c_1 = 8$. The solution of the initial-value problem is $3x^3\ln|x| + y^3 = 8x^3$.

13. Letting $y = ux$ we have

$$\begin{aligned}(x + uxe^u)\,dx - xe^u(u\,dx + x\,du) &= 0\\ dx - xe^u\,du &= 0\\ \frac{dx}{x} - e^u\,du &= 0\\ \ln|x| - e^u &= c\\ \ln|x| - e^{y/x} &= c.\end{aligned}$$

Using $y(1) = 0$ we find $c = -1$. The solution of the initial-value problem is $\ln|x| = e^{y/x} - 1$.

15. From $y' + \frac{1}{x}y = \frac{1}{x}y^{-2}$ and $w = y^3$ we obtain $\frac{dw}{dx} + \frac{3}{x}w = \frac{3}{x}$. An integrating factor is x^3 so that $x^3w = x^3 + c$ or $y^3 = 1 + cx^{-3}$.

17. From $y' + y = xy^4$ and $w = y^{-3}$ we obtain $\frac{dw}{dx} - 3w = -3x$. An integrating factor is e^{-3x} so that $e^{-3x}w = xe^{-3x} + \frac{1}{3}e^{-3x} + c$ or $y^{-3} = x + \frac{1}{3} + ce^{3x}$.

19. From $y' - \frac{1}{t}y = -\frac{1}{t^2}y^2$ and $w = y^{-1}$ we obtain $\frac{dw}{dt} + \frac{1}{t}w = \frac{1}{t^2}$. An integrating factor is t so that $tw = \ln t + c$ or $y^{-1} = \frac{1}{t}\ln t + \frac{c}{t}$. Writing this in the form $\frac{t}{y} = \ln t + c$, we see that the solution can also be expressed in the form $e^{t/y} = c_1t$.

21. From $y' - \frac{2}{x}y = \frac{3}{x^2}y^4$ and $w = y^{-3}$ we obtain $\frac{dw}{dx} + \frac{6}{x}w = -\frac{9}{x^2}$. An integrating factor is x^6 so that $x^6w = -\frac{9}{5}x^5 + c$ or $y^{-3} = -\frac{9}{5}x^{-1} + cx^{-6}$. If $y(1) = \frac{1}{2}$ then $c = \frac{49}{5}$ and $y^{-3} = -\frac{9}{5}x^{-1} + \frac{49}{5}x^{-6}$.

23. Let $u = x + y + 1$ so that $du/dx = 1 + dy/dx$. Then $\frac{du}{dx} - 1 = u^2$ or $\frac{1}{1+u^2}\,du = dx$. Thus $\tan^{-1}u = x + c$ or $u = \tan(x + c)$, and $x + y + 1 = \tan(x + c)$ or $y = \tan(x + c) - x - 1$.

25. Let $u = x + y$ so that $du/dx = 1 + dy/dx$. Then $\frac{du}{dx} - 1 = \tan^2 u$ or $\cos^2 u\,du = dx$. Thus $\frac{1}{2}u + \frac{1}{4}\sin 2u = x + c$ or $2u + \sin 2u = 4x + c_1$, and $2(x+y) + \sin 2(x+y) = 4x + c_1$ or $2y + \sin 2(x+y) = 2x + c_1$.

27. Let $u = y - 2x + 3$ so that $du/dx = dy/dx - 2$. Then $\frac{du}{dx} + 2 = 2 + \sqrt{u}$ or $\frac{1}{\sqrt{u}}\,du = dx$. Thus $2\sqrt{u} = x + c$ and $2\sqrt{y - 2x + 3} = x + c$.

29. Let $u = x + y$ so that $du/dx = 1 + dy/dx$. Then $\dfrac{du}{dx} - 1 = \cos u$ and $\dfrac{1}{1+\cos u}\,du = dx$. Now

$$\frac{1}{1+\cos u} = \frac{1-\cos u}{1-\cos^2 u} = \frac{1-\cos u}{\sin^2 u} = \csc^2 u - \csc u \cot u$$

so we have $\int(\csc^2 u - \csc u \cot u)du = \int dx$ and $-\cot u + \csc u = x + c$. Thus $-\cot(x+y) + \csc(x+y) = x + c$. Setting $x = 0$ and $y = \pi/4$ we obtain $c = \sqrt{2} - 1$. The solution is

$$\csc(x+y) - \cot(x+y) = x + \sqrt{2} - 1.$$

37. Write the differential equation as

$$\frac{dv}{dx} + \frac{1}{x}v = 32v^{-1},$$

and let $u = v^2$ or $v = u^{1/2}$. Then

$$\frac{dv}{dx} = \frac{1}{2}u^{-1/2}\frac{du}{dx},$$

and substituting into the differential equation, we have

$$\frac{1}{2}u^{-1/2}\frac{du}{dx} + \frac{1}{x}u^{1/2} = 32u^{-1/2} \qquad \text{or} \qquad \frac{du}{dx} + \frac{2}{x}u = 64.$$

The latter differential equation is linear with integrating factor $e^{\int(2/x)dx} = x^2$, so

$$\frac{d}{dx}\left[x^2 u\right] = 64x^2$$

and

$$x^2 u = \frac{64}{3}x^3 + c \qquad \text{or} \qquad v^2 = \frac{64}{3}x + \frac{c}{x^2}.$$

1. We identify $f(x, y) = 2x - 3y + 1$. Then, for $h = 0.1$,

$$y_{n+1} = y_n + 0.1(2x_n - 3y_n + 1) = 0.2x_n + 0.7y_n + 0.1,$$

and

$$y(1.1) \approx y_1 = 0.2(1) + 0.7(5) + 0.1 = 3.8$$

$$y(1.2) \approx y_2 = 0.2(1.1) + 0.7(3.8) + 0.1 = 2.98.$$

For $h = 0.05$,

$$y_{n+1} = y_n + 0.05(2x_n - 3y_n + 1) = 0.1x_n + 0.85y_n + 0.05,$$

and

$$y(1.05) \approx y_1 = 0.1(1) + 0.85(5) + 0.05 = 4.4$$

$$y(1.1) \approx y_2 = 0.1(1.05) + 0.85(4.4) + 0.05 = 3.895$$

$$y(1.15) \approx y_3 = 0.1(1.1) + 0.85(3.895) + 0.05 = 3.47075$$

$$y(1.2) \approx y_4 = 0.1(1.15) + 0.85(3.47075) + 0.05 = 3.11514.$$

3. Separating variables and integrating, we have

$$\frac{dy}{y} = dx \quad \text{and} \quad \ln|y| = x + c.$$

Thus $y = c_1e^x$ and, using $y(0) = 1$, we find $c = 1$, so $y = e^x$ is the solution of the initial-value problem.

h=0.1

x_n	y_n	Actual Value	Abs. Error	% Rel. Error
0.00	1.0000	1.0000	0.0000	0.00
0.10	1.1000	1.1052	0.0052	0.47
0.20	1.2100	1.2214	0.0114	0.93
0.30	1.3310	1.3499	0.0189	1.40
0.40	1.4641	1.4918	0.0277	1.86
0.50	1.6105	1.6487	0.0382	2.32
0.60	1.7716	1.8221	0.0506	2.77
0.70	1.9487	2.0138	0.0650	3.23
0.80	2.1436	2.2255	0.0820	3.68
0.90	2.3579	2.4596	0.1017	4.13
1.00	2.5937	2.7183	0.1245	4.58

h=0.05

x_n	y_n	Actual Value	Abs. Error	% Rel. Error
0.00	1.0000	1.0000	0.0000	0.00
0.05	1.0500	1.0513	0.0013	0.12
0.10	1.1025	1.1052	0.0027	0.24
0.15	1.1576	1.1618	0.0042	0.36
0.20	1.2155	1.2214	0.0059	0.48
0.25	1.2763	1.2840	0.0077	0.60
0.30	1.3401	1.3499	0.0098	0.72
0.35	1.4071	1.4191	0.0120	0.84
0.40	1.4775	1.4918	0.0144	0.96
0.45	1.5513	1.5683	0.0170	1.08
0.50	1.6289	1.6487	0.0198	1.20
0.55	1.7103	1.7333	0.0229	1.32
0.60	1.7959	1.8221	0.0263	1.44
0.65	1.8856	1.9155	0.0299	1.56
0.70	1.9799	2.0138	0.0338	1.68
0.75	2.0789	2.1170	0.0381	1.80
0.80	2.1829	2.2255	0.0427	1.92
0.85	2.2920	2.3396	0.0476	2.04
0.90	2.4066	2.4596	0.0530	2.15
0.95	2.5270	2.5857	0.0588	2.27
1.00	2.6533	2.7183	0.0650	2.39