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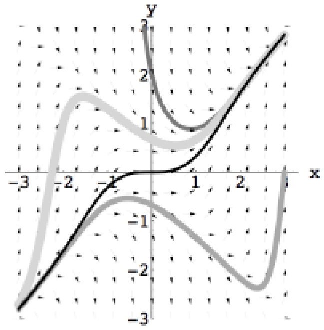

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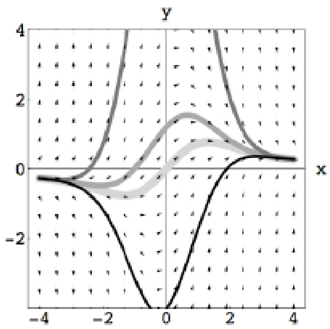

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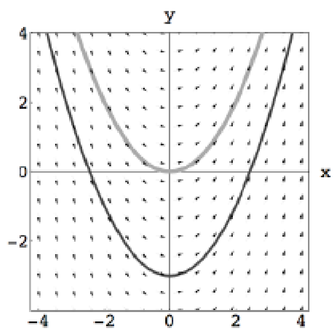

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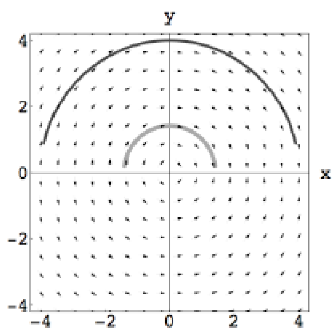

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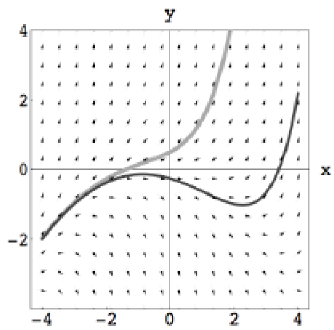

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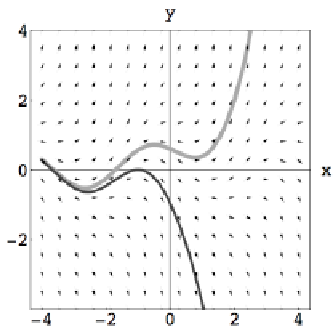

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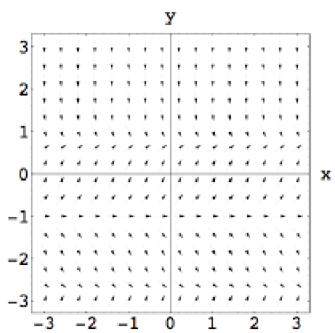
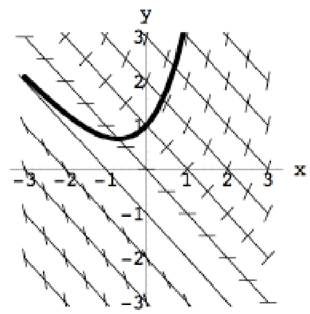
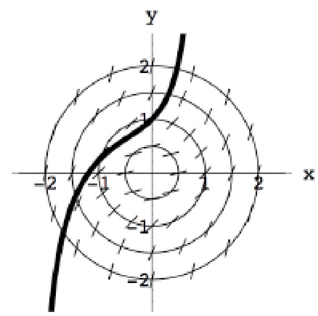


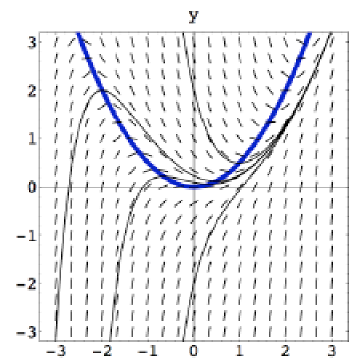
15. **(a)** The isoclines have the form $y = -x + c$, which are straight lines with slope -1.



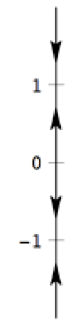
(b) The isoclines have the form $x^2 + y^2 = c$, which are circles centered at the origin.



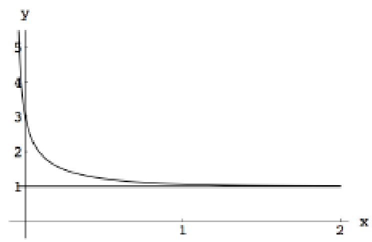
17. When $y < \frac{1}{2}x^2$, $y' = x^2 - 2y$ is positive and the portions of solution curves "outside" the nullcline parabola are increasing. When $y > \frac{1}{2}x^2$, $y' = x^2 - 2y$ is negative and the portions of the solution curves "inside" the nullcline parabola are decreasing.



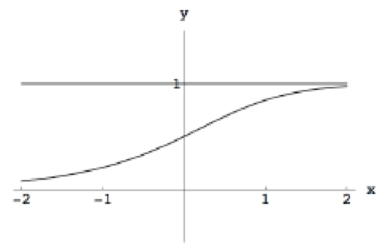
19. Writing the differential equation in the form $dy/dx = y(1-y)(1+y)$ we see that critical points are located at $y=-1$, $y=0$, and $y=1$. The phase portrait is shown at the right.



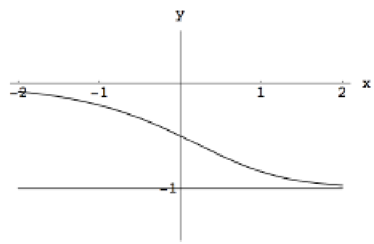
(a)



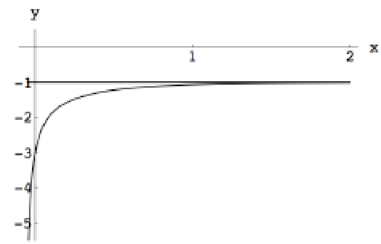
(b)



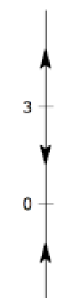
(c)



(d)

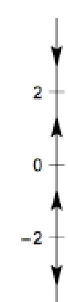


21. Solving $y^2 - 3y = y(y-3) = 0$ we obtain the critical points 0 and 3. From the phase portrait we see that 0 is asymptotically stable (attractor) and 3 is unstable (repeller).



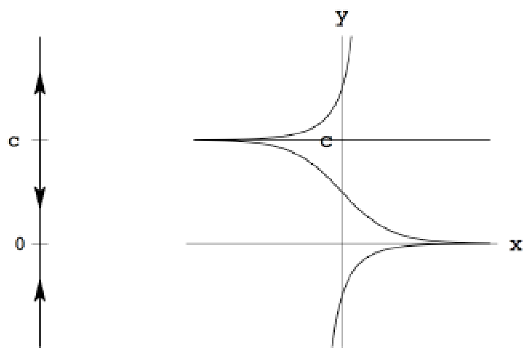
23. Solving $(y-2)^4 = 0$ we obtain the critical point 2. From the phase portrait we see that 2 is semi-stable.

25. Solving $y^2(4-y^2) = y^2(2-y)(2+y) = 0$ we obtain the critical points -2, 0, and 2. From the phase portrait we see that 2 is asymptotically stable (attractor), 0 is semi-stable, and -2 is unstable (repeller).

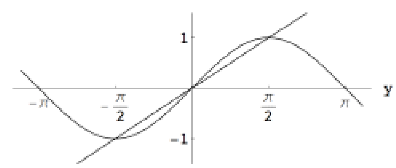


27. Solving $y\ln(y+2)=0$ we obtain the critical points -1 and 0. From the phase portrait we see that -1 is asymptotically stable (attractor) and 0 is unstable (repeller).

29. The critical points are 0 and c because the graph of $f(y)$ is 0 at these points. Since $f(y)>0$ for $y<0$ and $y>c$, the graph of the solution is increasing on $(-\infty,0)$ and (c,∞). Since $f(y)<0$ for $0<y<c$, the graph of the solution is decreasing on $(0,c)$.



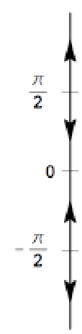
31. From the graphs of $z=\pi/2$ and $z=\sin y$ we see that $(\pi/2)y-\sin y=0$ has only three solutions. By inspection we see that the critical points are $-\pi/2$, 0, and $\pi/2$.



From the graph at the right we see that

$$\frac{2}{\pi}y-\sin y\begin{cases}<0 & \text{for} \quad y<-\pi/2\\ >0 & \text{for} \quad y>\pi/2\end{cases}$$

$$\frac{2}{\pi}y-\sin y\begin{cases}>0 & \text{for} \quad -\pi/2<y<0\\ <0 & \text{for} \quad 0<y<\pi/2.\end{cases}$$



This enables us to construct the phase portrait shown at the right. From this portrait we see that $\pi/2$ and $-\pi/2$ are unstable (repellers), and 0 is asymptotically stable (attractor).

33. Recall that for $dy/dx = f(y)$ we are assuming that f and f' are continuous functions of y on some interval I. Now suppose that the graph of a nonconstant solution of the differential equation crosses the line $y = c$. If the point of intersection is taken as an initial condition we have two distinct solutions of the initial-value problem. This violates uniqueness, so the graph of any nonconstant solution must lie entirely on one side of any equilibrium solution. Since f is continuous it can only change signs at a point where it is 0. But this is a critical point. Thus, $f(y)$ is completely positive or completely negative in each region R_i. If $y(x)$ is oscillatory or has a relative extremum, then it must have a horizontal tangent line at some point (x_0, y_0). In this case y_0 would be a critical point of the differential equation, but we saw above that the graph of a nonconstant solution cannot intersect the graph of the equilibrium solution $y = y_0$.

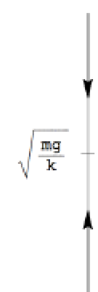
35. Assuming the existence of the second derivative, points of inflection of $y(x)$ occur where $y''(x) = 0$. From $dy/dx = f(y)$ we have $d^2y/dx^2 = f'(y)\,dy/dx$. Thus, the y-coordinate of a point of inflection can be located by solving $f'(y) = 0$. (Points where $dy/dx = 0$ correspond to constant solutions of the differential equation.)

37. If (1) in the text has no critical points it has no constant solutions. The solutions have neither an upper nor lower bound. Since solutions are monotonic, every solution assumes all real values.

39. The only critical point of the autonomous differential equation is the positive number h/k. A phase portrait shows that this point is unstable, so h/k is a repeller. For any initial condition $P(0) = P_0 < h/k, dP/dt < 0$, which means $P(t)$ is monotonic decreasing and so the graph of $P(t)$ must cross the t-axis or the line $P = 0$ at some time $t_1 > 0$. But $P(t_1) = 0$ means the population is extinct at time t_1.

41. Writing the differential equation in the form

$$\frac{dv}{dt} = \frac{k}{m}\left(\frac{mg}{k} - v^2\right) = \frac{k}{m}\left(\sqrt{\frac{mg}{k}} - v\right)\left(\sqrt{\frac{mg}{k}} + v\right)$$



we see that the only physically meaningful critical point is $\sqrt{mg/k}$.

From the phase portrait we see that $\sqrt{mg/k}$ is an asymptotically stable critical point. Thus, $\lim_{t\to\infty} v = \sqrt{mg/k}$.