1. $\mathscr{L}^{-1}\left\{\frac{1}{s^3}\right\} = \frac{1}{2}\mathscr{L}^{-1}\left\{\frac{2}{s^3}\right\} = \frac{1}{2}t^2$

3. $\mathscr{L}^{-1}\left\{\frac{1}{s^2} - \frac{48}{s^5}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s^2} - \frac{48}{24}\cdot\frac{4!}{s^5}\right\} = t - 2t^4$

5. $\mathscr{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s} + 3\cdot\frac{1}{s^2} + \frac{3}{2}\cdot\frac{2}{s^3} + \frac{1}{6}\cdot\frac{3!}{s^4}\right\} = 1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3$

7. $\mathscr{L}^{-1}\left\{\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s-2}\right\} = t - 1 + e^{2t}$

9. $\mathscr{L}^{-1}\left\{\frac{1}{4s+1}\right\} = \frac{1}{4}\mathscr{L}^{-1}\left\{\frac{1}{s+1/4}\right\} = \frac{1}{4}e^{-t/4}$

11. $\mathscr{L}^{-1}\left\{\frac{5}{s^2+49}\right\} = \mathscr{L}^{-1}\left\{\frac{5}{7}\cdot\frac{7}{s^2+49}\right\} = \frac{5}{7}\sin 7t$

13. $\mathscr{L}^{-1}\left\{\frac{4s}{4s^2+1}\right\} = \mathscr{L}^{-1}\left\{\frac{s}{s^2+1/4}\right\} = \cos\frac{1}{2}t$

15. $\mathscr{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\} = \mathscr{L}^{-1}\left\{2\cdot\frac{s}{s^2+9} - 2\cdot\frac{3}{s^2+9}\right\} = 2\cos 3t - 2\sin 3t$

17. $\mathscr{L}^{-1}\left\{\frac{1}{s^2+3s}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{3}\cdot\frac{1}{s} - \frac{1}{3}\cdot\frac{1}{s+3}\right\} = \frac{1}{3} - \frac{1}{3}e^{-3t}$

19. $\mathscr{L}^{-1}\left\{\frac{s}{s^2+2s-3}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{4}\cdot\frac{1}{s-1} + \frac{3}{4}\cdot\frac{1}{s+3}\right\} = \frac{1}{4}e^t + \frac{3}{4}e^{-3t}$

21. $\mathscr{L}^{-1}\left\{\frac{0.9s}{(s-0.1)(s+0.2)}\right\} = \mathscr{L}^{-1}\left\{(0.3)\cdot\frac{1}{s-0.1} + (0.6)\cdot\frac{1}{s+0.2}\right\} = 0.3e^{0.1t} + 0.6e^{-0.2t}$

23. $\mathscr{L}^{-1}\left\{\frac{s}{(s-2)(s-3)(s-6)}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{2}\cdot\frac{1}{s-2} - \frac{1}{s-3} + \frac{1}{2}\cdot\frac{1}{s-6}\right\} = \frac{1}{2}e^{2t} - e^{3t} + \frac{1}{2}e^{6t}$

25. $\mathscr{L}^{-1}\left\{\frac{1}{s^3+5s}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s(s^2+5)}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{5}\cdot\frac{1}{s} - \frac{1}{5}\frac{s}{s^2+5}\right\} = \frac{1}{5} - \frac{1}{5}\cos\sqrt{5}t$

27. $\mathscr{L}^{-1}\left\{\frac{2s-4}{(s^2+s)(s^2+1)}\right\} = \mathscr{L}^{-1}\left\{\frac{2s-4}{s(s+1)(s^2+1)}\right\} = \mathscr{L}^{-1}\left\{-\frac{4}{s} + \frac{3}{s+1} + \frac{s}{s^2+1} + \frac{3}{s^2+1}\right\}$

$$= -4 + 3e^{-t} + \cos t + 3\sin t$$

29. $$\mathscr{L}^{-1}\left\{\frac{1}{(s^2+1)(s^2+4)}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{3}\cdot\frac{1}{s^2+1} - \frac{1}{3}\cdot\frac{1}{s^2+4}\right\}$$
$$= \mathscr{L}^{-1}\left\{\frac{1}{3}\cdot\frac{1}{s^2+1} - \frac{1}{6}\cdot\frac{2}{s^2+4}\right\}$$
$$= \frac{1}{3}\sin t - \frac{1}{6}\sin 2t$$

31. The Laplace transform of the initial-value problem is

$$s\,\mathscr{L}\{y\} - y(0) - \mathscr{L}\{y\} = \frac{1}{s}.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\} = -\frac{1}{s} + \frac{1}{s-1}.$$

Thus

$$y = -1 + e^t.$$

33. The Laplace transform of the initial-value problem is

$$s\mathscr{L}\{y\} - y(0) + 6\mathscr{L}\{y\} = \frac{1}{s-4}.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\} = \frac{1}{(s-4)(s+6)} + \frac{2}{s+6} = \frac{1}{10}\cdot\frac{1}{s-4} + \frac{19}{10}\cdot\frac{1}{s+6}.$$

Thus

$$y = \frac{1}{10}e^{4t} + \frac{19}{10}e^{-6t}.$$

35. The Laplace transform of the initial-value problem is

$$s^2\mathscr{L}\{y\} - sy(0) - y'(0) + 5\left[s\,\mathscr{L}\{y\} - y(0)\right] + 4\,\mathscr{L}\{y\} = 0.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\} = \frac{s+5}{s^2+5s+4} = \frac{4}{3}\,\frac{1}{s+1} - \frac{1}{3}\,\frac{1}{s+4}.$$

Thus

$$y = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}.$$

37. The Laplace transform of the initial-value problem is

$$s^2\mathscr{L}\{y\} - sy(0) + \mathscr{L}\{y\} = \frac{2}{s^2+2}.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\} = \frac{2}{(s^2+1)(s^2+2)} + \frac{10s}{s^2+1} = \frac{10s}{s^2+1} + \frac{2}{s^2+1} - \frac{2}{s^2+2}.$$

Thus

$$y = 10\cos t + 2\sin t - \sqrt{2}\sin\sqrt{2}\,t.$$

39. The Laplace transform of the initial-value problem is

$$2\left[s^3\mathscr{L}\{y\} - s^2y(0) - sy'(0) - y''(0)\right] + 3[s^2\mathscr{L}\{y\} - sy(0) - y'(0)] - 3[s\mathscr{L}\{y\} - y(0)] - 2\mathscr{L}\{y\} = \frac{1}{s+1}.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\} = \frac{2s+3}{(s+1)(s-1)(2s+1)(s+2)} = \frac{1}{2}\frac{1}{s+1} + \frac{5}{18}\frac{1}{s-1} - \frac{8}{9}\frac{1}{s+1/2} + \frac{1}{9}\frac{1}{s+2}.$$

Thus

$$y = \frac{1}{2}e^{-t} + \frac{5}{18}e^{t} - \frac{8}{9}e^{-t/2} + \frac{1}{9}e^{-2t}.$$

41. The Laplace transform of the initial-value problem is

$$s\mathscr{L}\{y\} + \mathscr{L}\{y\} = \frac{s+3}{s^2+6s+13}.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\} = \frac{s+3}{(s+1)(s^2+6s+13)} = \frac{1}{4}\cdot\frac{1}{s+1} - \frac{1}{4}\cdot\frac{s+1}{s^2+6s+13}$$

$$= \frac{1}{4}\cdot\frac{1}{s+1} - \frac{1}{4}\left(\frac{s+3}{(s+3)^2+4} - \frac{2}{(s+3)^2+4}\right).$$

Thus

$$y = \frac{1}{4}e^{-t} - \frac{1}{4}e^{-3t}\cos 2t + \frac{1}{4}e^{-3t}\sin 2t.$$