1. $$\mathscr{L}\{f(t)\} = \int_0^1 -e^{-st}dt + \int_1^\infty e^{-st}dt = \frac{1}{s}e^{-st}\Big|_0^1 - \frac{1}{s}e^{-st}\Big|_1^\infty$$
$$= \frac{1}{s}e^{-s} - \frac{1}{s} - \left(0 - \frac{1}{s}e^{-s}\right) = \frac{2}{s}e^{-s} - \frac{1}{s}, \quad s > 0$$

3. $$\mathscr{L}\{f(t)\} = \int_0^1 te^{-st}dt + \int_1^\infty e^{-st}dt = \left(-\frac{1}{s}te^{-st} - \frac{1}{s^2}e^{-st}\right)\Big|_0^1 - \frac{1}{s}e^{-st}\Big|_1^\infty$$
$$= \left(-\frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-s}\right) - \left(0 - \frac{1}{s^2}\right) - \frac{1}{s}(0 - e^{-s}) = \frac{1}{s^2}(1 - e^{-s}), \quad s > 0$$

5. $$\mathscr{L}\{f(t)\} = \int_0^\pi (\sin t)e^{-st}dt = \left(-\frac{s}{s^2+1}e^{-st}\sin t - \frac{1}{s^2+1}e^{-st}\cos t\right)\Big|_0^\pi$$
$$= \left(0 + \frac{1}{s^2+1}e^{-\pi s}\right) - \left(0 - \frac{1}{s^2+1}\right) = \frac{1}{s^2+1}(e^{-\pi s} + 1), \quad s > 0$$

7. $$f(t) = \begin{cases} 0, & 0 < t < 1 \\ t, & t > 1 \end{cases}$$
$$\mathscr{L}\{f(t)\} = \int_1^\infty te^{-st}\,dt = \left(-\frac{1}{s}te^{-st} - \frac{1}{s^2}\,e^{-st}\right)\Big|_1^\infty = \frac{1}{s}\,e^{-s} + \frac{1}{s^2}\,e^{-s}, \quad s > 0$$

9. The function is $f(t) = \begin{cases} 1-t, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$ so
$$\mathscr{L}\{f(t)\} = \int_0^1 (1-t)e^{-st}\,dt + \int_1^\infty 0e^{-st}\,dt = \int_0^1 (1-t)e^{-st}\,dt = \left(-\frac{1}{s}(1-t)e^{-st} + \frac{1}{s^2}\,e^{-st}\right)\Big|_0^1$$
$$= \frac{1}{s^2}\,e^{-s} + \frac{1}{s} - \frac{1}{s^2}, \quad s > 0$$

11. $$\mathscr{L}\{f(t)\} = \int_0^\infty e^{t+7}e^{-st}dt = e^7\int_0^\infty e^{(1-s)t}dt = \frac{e^7}{1-s}e^{(1-s)t}\Big|_0^\infty = 0 - \frac{e^7}{1-s} = \frac{e^7}{s-1}, \quad s > 1$$

13. $$\mathscr{L}\{f(t)\} = \int_0^\infty te^{4t}e^{-st}dt = \int_0^\infty te^{(4-s)t}dt = \left(\frac{1}{4-s}te^{(4-s)t} - \frac{1}{(4-s)^2}e^{(4-s)t}\right)\Big|_0^\infty$$
$$= \frac{1}{(4-s)^2}, \quad s > 4$$

15. $$\mathscr{L}\{f(t)\} = \int_0^\infty e^{-t}(\sin t)e^{-st}dt = \int_0^\infty (\sin t)e^{-(s+1)t}dt$$
$$= \left(\frac{-(s+1)}{(s+1)^2+1}e^{-(s+1)t}\sin t - \frac{1}{(s+1)^2+1}e^{-(s+1)t}\cos t\right)\Big|_0^\infty$$
$$= \frac{1}{(s+1)^2+1} = \frac{1}{s^2+2s+2}, \quad s > -1$$

17. $$\begin{aligned}\mathscr{L}\{f(t)\} &= \int_0^\infty t(\cos t)e^{-st}dt \\ &= \left[\left(-\frac{st}{s^2+1} - \frac{s^2-1}{(s^2+1)^2}\right)(\cos t)e^{-st} + \left(\frac{t}{s^2+1} + \frac{2s}{(s^2+1)^2}\right)(\sin t)e^{-st}\right]_0^\infty \\ &= \frac{s^2-1}{(s^2+1)^2}, \quad s > 0\end{aligned}$$

19. $\mathscr{L}\{2t^4\} = 2\dfrac{4!}{s^5}$

21. $\mathscr{L}\{4t-10\} = \dfrac{4}{s^2} - \dfrac{10}{s}$

23. $\mathscr{L}\{t^2+6t-3\} = \dfrac{2}{s^3} + \dfrac{6}{s^2} - \dfrac{3}{s}$

25. $\mathscr{L}\{t^3+3t^2+3t+1\} = \dfrac{3!}{s^4} + 3\dfrac{2}{s^3} + \dfrac{3}{s^2} + \dfrac{1}{s}$

27. $\mathscr{L}\{1+e^{4t}\} = \dfrac{1}{s} + \dfrac{1}{s-4}$

29. $\mathscr{L}\{1+2e^{2t}+e^{4t}\} = \dfrac{1}{s} + \dfrac{2}{s-2} + \dfrac{1}{s-4}$

31. $\mathscr{L}\{4t^2 - 5\sin 3t\} = 4\dfrac{2}{s^3} - 5\dfrac{3}{s^2+9}$

33. $\mathscr{L}\{\sinh kt\} = \dfrac{1}{2}\mathscr{L}\{e^{kt} - e^{-kt}\} = \dfrac{1}{2}\left[\dfrac{1}{s-k} - \dfrac{1}{s+k}\right] = \dfrac{k}{s^2-k^2}$

35. $\mathscr{L}\{e^t \sinh t\} = \mathscr{L}\left\{e^t\,\dfrac{e^t - e^{-t}}{2}\right\} = \mathscr{L}\left\{\dfrac{1}{2}e^{2t} - \dfrac{1}{2}\right\} = \dfrac{1}{2(s-2)} - \dfrac{1}{2s}$

37. $\mathscr{L}\{\sin 2t \cos 2t\} = \mathscr{L}\left\{\dfrac{1}{2}\sin 4t\right\} = \dfrac{2}{s^2+16}$

39. From the addition formula for the sine function, $\sin(4t+5) = (\sin 4t)(\cos 5) + (\cos 4t)(\sin 5)$ so

$$\begin{aligned}\mathscr{L}\{\sin(4t+5)\} &= (\cos 5)\,\mathscr{L}\{\sin 4t\} + (\sin 5)\,\mathscr{L}\{\cos 4t\} \\ &= (\cos 5)\,\frac{4}{s^2+16} + (\sin 5)\,\frac{s}{s^2+16} \\ &= \frac{4\cos 5 + (\sin 5)s}{s^2+16}.\end{aligned}$$

41. **(a)** Using integration by parts for $\alpha > 0$,

$$\Gamma(\alpha+1) = \int_0^\infty t^\alpha e^{-t}\,dt = -t^\alpha e^{-t}\Big|_0^\infty + \alpha\int_0^\infty t^{\alpha-1}e^{-t}\,dt = \alpha\Gamma(\alpha).$$

(b) Let $u = st$ so that $du = s\,dt$. Then

$$\mathscr{L}\{t^\alpha\} = \int_0^\infty e^{-st}t^\alpha dt = \int_0^\infty e^{-u}\left(\frac{u}{s}\right)^\alpha \frac{1}{s}\,du = \frac{1}{s^{\alpha+1}}\Gamma(\alpha+1), \quad \alpha > -1.$$