1. Second order; linear

3. Fourth order; linear

5. Second order; nonlinear because of $(dy/dx)^2$ or $\sqrt{1+(dy/dx)^2}$

7. Third order; linear

9. Writing the differential equation in the form $x(dy/dx)+y^2=1$, we see that it is nonlinear in y because of y^2. However, writing it in the form $(y^2-1)(dx/dy)+x=0$, we see that it is linear in x.

11. From $y=e^{-x/2}$ we obtain $y'=-\frac{1}{2}e^{-x/2}$. Then $2y'+y=-e^{-x/2}+e^{-x/2}=0$.

13. From $y=e^{3x}\cos 2x$ we obtain $y'=3e^{3x}\cos 2x-2e^{3x}\sin 2x$ and $y''=5e^{3x}\cos 2x-12e^{3x}\sin 2x$, so that $y''-6y'+13y=0$.

15. The domain of the function, found by solving $x+2\geq 0$, is $[-2,\infty)$. From $y'=1+2(x+2)^{-1/2}$ we have

$$\begin{aligned}(y-x)y' &= (y-x)[1+(2(x+2)^{-1/2}]\\ &= y-x+2(y-x)(x+2)^{-1/2}\\ &= y-x+2[x+4(x+2)^{1/2}-x](x+2)^{-1/2}\\ &= y-x+8(x+2)^{1/2}(x+2)^{-1/2}=y-x+8.\end{aligned}$$

An interval of definition for the solution of the differential equation is $(-2,\infty)$ because y' is not defined at $x=-2$.

17. The domain of the function is $\{x \mid 4-x^2\neq 0\}$ or $\{x \mid x\neq -2 \text{ or } x\neq 2\}$. From $y'=2x/(4-x^2)^2$ we have

$$y'=2x\left(\frac{1}{4-x^2}\right)^2=2xy.$$

An interval of definition for the solution of the differential equation is $(-2,2)$. Other intervals are $(-\infty,-2)$ and $(2,\infty)$.

19. Writing $\ln(2X-1)-\ln(X-1)=t$ and differentiating implicitly we obtain

$$\frac{2}{2X-1}\frac{dX}{dt}-\frac{1}{X-1}\frac{dX}{dt}=1$$

$$\left(\frac{2}{2X-1}-\frac{1}{X-1}\right)\frac{dX}{dt}=1$$

$$\frac{2X-2-2X+1}{(2X-1)(X-1)}\frac{dX}{dt}=1$$

$$\frac{dX}{dt}=-(2X-1)(X-1)=(X-1)(1-2X).$$

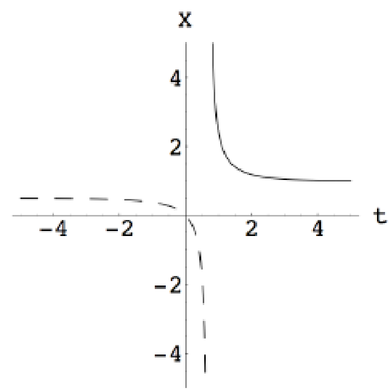
Exponentiating both sides of the implicit solution we obtain

$$\frac{2X-1}{X-1}=e^t$$

$$2X-1=Xe^t-e^t$$

$$(e^t-1)=(e^t-2)X$$

$$X=\frac{e^t-1}{e^t-2}.$$



Solving $e^t-2=0$ we get $t=\ln 2$. Thus, the solution is defined on $(-\infty,\ln 2)$ or on $(\ln 2,\infty)$. The graph of the solution defined on $(-\infty,\ln 2)$ is dashed, and the graph of the solution defined on $(\ln 2,\infty)$ is solid.

21. Differentiating $P=c_1e^t/\left(1+c_1e^t\right)$ we obtain

$$\frac{dP}{dt}=\frac{\left(1+c_1e^t\right)c_1e^t-c_1e^t\cdot c_1e^t}{(1+c_1e^t)^2}=\frac{c_1e^t}{1+c_1e^t}\frac{\left[\left(1+c_1e^t\right)-c_1e^t\right]}{1+c_1e^t}$$

$$=\frac{c_1e^t}{1+c_1e^t}\left[1-\frac{c_1e^t}{1+c_1e^t}\right]=P(1-P).$$

23. From $y=c_1e^{2x}+c_2xe^{2x}$ we obtain $\dfrac{dy}{dx}=(2c_1+c_2)e^{2x}+2c_2xe^{2x}$ and $\dfrac{d^2y}{dx^2}=(4c_1+4c_2)e^{2x}+4c_2xe^{2x}$, so that

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=(4c_1+4c_2-8c_1-4c_2+4c_1)e^{2x}+(4c_2-8c_2+4c_2)xe^{2x}=0.$$

25. From $y=\begin{cases}-x^2, & x<0\\ x^2, & x\ge 0\end{cases}$ we obtain $y'=\begin{cases}-2x, & x<0\\ 2x, & x\ge 0\end{cases}$ so that $xy'-2y=0$.

27. From $y = e^{mx}$ we obtain $y' = me^{mx}$. Then $y' + 2y = 0$ implies

$$me^{mx} + 2e^{mx} = (m+2)e^{mx} = 0.$$

Since $e^{mx} > 0$ for all x, $m = -2$. Thus $y = e^{-2x}$ is a solution.

29. From $y = e^{mx}$ we obtain $y' = me^{mx}$ and $y'' = m^2e^{mx}$. Then $y'' - 5y' + 6y = 0$ implies

$$m^2e^{mx} - 5me^{mx} + 6e^{mx} = (m-2)(m-3)e^{mx} = 0.$$

Since $e^{mx} > 0$ for all x, $m = 2$ and $m = 3$. Thus $y = e^{2x}$ and $y = e^{3x}$ are solutions.

31. From $y = x^m$ we obtain $y' = mx^{m-1}$ and $y'' = m(m-1)x^{m-2}$. Then $xy'' + 2y' = 0$ implies

$$xm(m-1)x^{m-2} + 2mx^{m-1} = [m(m-1) + 2m]x^{m-1} = (m^2 + m)x^{m-1}$$

$$= m(m+1)x^{m-1} = 0.$$

Since $x^{m-1} > 0$ for $x > 0$, $m = 0$ and $m = -1$. Thus $y = 1$ and $y = x^{-1}$ are solutions.

In Problems 33–36 we substitute $y = c$ into the differential equations and use $y' = 0$ and $y'' = 0$

33. Solving $5c = 10$ we see that $y = 2$ is a constant solution.

In Problems 33–36 we substitute $y = c$ into the differential equations and use $y' = 0$ and $y'' = 0$

35. Since $1/(c-1) = 0$ has no solutions, the differential equation has no constant solutions.

37. From $x = e^{-2t} + 3e^{6t}$ and $y = -e^{-2t} + 5e^{6t}$ we obtain

$$\frac{dx}{dt} = -2e^{-2t} + 18e^{6t} \quad \text{and} \quad \frac{dy}{dt} = 2e^{-2t} + 30e^{6t}.$$

Then

$$x + 3y = (e^{-2t} + 3e^{6t}) + 3(-e^{-2t} + 5e^{6t}) = -2e^{-2t} + 18e^{6t} = \frac{dx}{dt}$$

and

$$5x + 3y = 5(e^{-2t} + 3e^{6t}) + 3(-e^{-2t} + 5e^{6t}) = 2e^{-2t} + 30e^{6t} = \frac{dy}{dt}.$$

47. Differentiating $(x^3 + y^3)/xy = 3c$ we obtain

$$\frac{xy(3x^2 + 3y^2y') - (x^3 + y^3)(xy' + y)}{x^2y^2} = 0$$

$$3x^3y + 3xy^3y' - x^4y' - x^3y - xy^3y' - y^4 = 0$$

$$(3xy^3 - x^4 - xy^3)y' = -3x^3y + x^3y + y^4$$

$$y' = \frac{y^4 - 2x^3y}{2xy^3 - x^4} = \frac{y(y^3 - 2x^3)}{x(2y^3 - x^3)}.$$

49. The derivatives of the functions are $\phi_1'(x) = -x/\sqrt{25-x^2}$ and $\phi_2'(x) = x/\sqrt{25-x^2}$, neither of which is defined at $x = \pm 5$.

51. For the first-order differential equation integrate $f(x)$. For the second-order differential equation integrate twice. In the latter case we get $y = \int(\int f(x)dx)dx + c_1x + c_2$.

53. The differential equation $yy' - xy = 0$ has normal form $dy/dx = x$. These are not equivalent because $y = 0$ is a solution of the first differential equation but not a solution of the second.

55. **(a)** Since e^{-x^2} is positive for all values of x, $dy/dx > 0$ for all x, and a solution, $y(x)$, of the differential equation must be increasing on any interval.

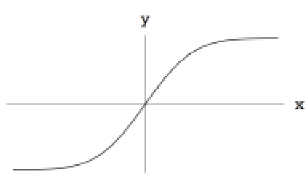
(b) $\lim_{x\to-\infty} \frac{dy}{dx} = \lim_{x\to-\infty} e^{-x^2} = 0$ and $\lim_{x\to\infty} \frac{dy}{dx} = \lim_{x\to\infty} e^{-x^2} = 0$. Since dy/dx approaches 0 as x approaches $-\infty$ and ∞, the solution curve has horizontal asymptotes to the left and to the right.

(c) To test concavity we consider the second derivative

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(e^{-x^2}\right) = -2xe^{-x^2}.$$

Since the second derivative is positive for $x < 0$ and negative for $x > 0$, the solution curve is concave up on $(-\infty, 0)$ and concave down on $(0, \infty)$. x

(d)



57. **(a)** The derivative of a constant solution is 0, so solving $y(a - by) = 0$ we see that $y = 0$ and $y = a/b$ are constant solutions.

(b) A solution is increasing where $dy/dx = y(a - by) = by(a/b - y) > 0$ or $0 < y < a/b$. A solution is decreasing where $dy/dx = by(a/b - y) < 0$ or $y < 0$ or $y > a/b$.

(c) Using implicit differentiation we compute

$$\frac{d^2y}{dx^2} = y(-by') + y'(a - by) = y'(a - 2by).$$

Solving $d^2y/dx^2 = 0$ we obtain $y = a/2b$. Since $d^2y/dx^2 > 0$ for $0 < y < a/2b$ and $d^2y/dx^2 < 0$ for $a/2b < y < a/b$, the graph of $y = \phi(x)$ has a point of inflection at $y = a/2b$.

(d)