1.1 and 1.2

Show that each statement is true.

13. If \( DE \) has endpoints \( D(-1, 6) \) and \( E(3, -2) \), then the midpoint \( M \) of \( DE \) lies in Quadrant I.

\[
M\left(\frac{-1 + 3}{2}, \frac{6 + (-2)}{2}\right) = M(1, 2)
\]

So \( M \) lies in Quadrant I, since the x- and y-coordinates are both positive.

Show that each statement is true.

15. If \( JK \) has endpoints \( J(-2, 3) \) and \( K(6, 5) \), and \( LN \) has endpoints \( L(0, 7) \) and \( N(4, 1) \), then \( JK \) and \( LN \) have the same midpoint.

\[
M_{\text{JK}}\left(\frac{-2 + 6}{2}, \frac{3 + 5}{2}\right) = M_{\text{JK}}(2, 4)
\]

\[
M_{\text{LN}}\left(\frac{0 + 4}{2}, \frac{7 + 1}{2}\right) = M_{\text{LN}}(2, 4)
\]

Both midpoints have the same coordinates, so the segments have the same midpoint.

23. Multi-Step The sign shows distances from a rest stop to the exits for different towns along a straight section of highway. The state department of transportation is planning to build a new exit to Freestone at the midpoint of the exits for Roseville and Edgewood. When the new exit is built, what will be the distance from the exit for Midtown to the exit for Freestone?

The distance from the Roseville to Edgewood exits is \( 59 - 35 = 24 \) mi, so the distance from the Roseville to Freestone exits will be \( \frac{1}{2} \cdot 24 = 12 \) mi. The distance from the Midtown to Roseville exits is \( 35 - 17 = 18 \) mi, so the distance from the Midtown to Freestone exits will be \( 18 + 12 = 30 \) mi.

24. On a town map, each unit of the coordinate plane represents 1 mile. Three branches of a bank are located at \( A(-3, 1) \), \( B(2, 3) \), and \( C(4, -1) \). A bank employee drives from Branch A to Branch B and then drives halfway to Branch C before getting stuck in traffic. What is the minimum total distance the employee may have driven before getting stuck in traffic? Round to the nearest tenth of a mile.

The minimum total distance occurs when the employee drives along a straight line from \( A \) to \( B \) and from \( B \) to the midpoint of \( BC \).

The midpoint \( N \) of \( BC \) is \( N(3, 1) \).

\[
AB = \sqrt{29}, BN = \sqrt{5}, AB + BN = \sqrt{29} + \sqrt{5} \approx 7.6.
\]

The minimum total distance the employee may have driven is 7.6 miles.

25. A city planner designs a park that is a quadrilateral with vertices at \( J(-3, 1) \), \( K(1, 3) \), \( L(5, -1) \), and \( M(-1, -3) \). There is an entrance to the park at the midpoint of each side of the park. A straight path connects each entrance to the entrance on the opposite side. Assuming each unit of the coordinate plane represents 10 meters, what is the total length of the paths to the nearest meter?

Midpoint \( P \) of \( JK \) is \( P(-1, 1) \), The paths are \( PR \) and \( SQ \).

Midpoint \( Q \) of \( KL \) is \( Q(3, 1) \), \( PR = \sqrt{25} = 5 \)

Midpoint \( R \) of \( LM \) is \( R(2, -2) \), \( SQ = \sqrt{29} \)

Midpoint \( S \) of \( MJ \) is \( S(-2, -1) \).

The total length is \( \sqrt{29} + 5 \approx 10.39 \), which represents 103.9 meters.

The total length of the paths is approximately 104 meters.
26. Communicate Mathematical Ideas A video game designer places an ant hill at the origin of a coordinate plane. A red ant leaves the ant hill and moves along a straight line to (1, 1), while a black ant leaves the ant hill and moves along a straight line to (−1, −1). Next, the red ant moves to (2, 2), while the black ant moves to (−2, −2). Then the red ant moves to (3, 3), while the black ant moves to (−3, −3), and so on. Explain why the red ant and the black ant are always the same distance from the ant hill.

At any given moment, the red ant’s coordinates may be written as \((a, a)\) where \(a > 0\). The red ant’s distance from the ant hill is \(\sqrt{(a - 0)^2 + (a - 0)^2} = \sqrt{2a^2} = a\sqrt{2}\).

The black ant’s coordinates may be written as \((-a, -a)\) and the black ant’s distance from the ant hill is \(\sqrt{(-a - 0)^2 + (-a - 0)^2} = \sqrt{2a^2} = a\sqrt{2}\). This shows both ants are always \(a\sqrt{2}\) units from the ant hill.

Use a compass and straightedge to construct a copy of each angle.

1.  
   ![Angle 1]

2.  
   ![Angle 2]

3.  
   ![Angle 3]

Draw an angle with the given name.

4. \(\angle JWT\)

   ![Angle JWT]

5. \(\angle NBQ\)

   ![Angle NBQ]

Name each angle in as many different ways as possible.

6.  
   ![Angle 6]

\(\angle W, \angle ZWX, \angle XWZ, \text{ and } \angle 1\)

7.  
   ![Angle 7]

\(\angle L, \angle GLJ, \angle JLG, \text{ and } \angle 2\)
Use a compass and straightedge to construct the bisector of the given angle. Check that the measure of each of the new angles is one-half the measure of the given angle.

12. 

13. 

14. 

Use the Angle Addition Postulate to find the measure of each angle.

15. \( \angle BXC \)

\[
\begin{align*}
\text{m} \angle AXB + \text{m} \angle BXC &= \text{m} \angle AXC \\
40^\circ + \text{m} \angle BXC &= 70^\circ \\
\text{m} \angle BXC &= 30^\circ 
\end{align*}
\]

16. \( \angle BXE \)

\[
\begin{align*}
\text{m} \angle EXF + \text{m} \angle BXE &= \text{m} \angle BXF \\
30^\circ + \text{m} \angle BXE &= 140^\circ \\
\text{m} \angle BXE &= 110^\circ 
\end{align*}
\]

Use a compass and straightedge to copy each angle onto a separate piece of paper. Then use paper folding to construct the angle bisector.

18. 

19. Use a compass and straightedge to construct an angle whose measure is \( \text{m} \angle A + \text{m} \angle B \). Use a protractor to check your construction.

\( A \)

\( B \)
20. Find the value of $x$, given that $\angle PQS = 112^\circ$.

\[
m\angle PQR + m\angle RQS = m\angle PQS
\]
\[
72 + 10x = 112
\]
\[
x = 4
\]

21. Find the value of $y$, given that $\angle KLM = 135^\circ$.

\[
m\angle KLN + m\angle NLM = m\angle KLM
\]
\[
47 + 16y = 135
\]
\[
y = 5.5
\]

22. Multi-Step  The figure shows a map of five streets that meet at Concord Circle. The measure of the angle formed by Melville Road and Emerson Avenue is $118^\circ$. The measure of the angle formed by Emerson Avenue and Thoreau Street is $134^\circ$. Hawthorne Lane bisects the angle formed by Melville Road and Emerson Avenue. Dickinson Drive bisects the angle formed by Emerson Avenue and Thoreau Street. What is the measure of the angle formed by Hawthorne Lane and Dickinson Drive? Explain your reasoning.

The measure of the angle formed by Melville and Emerson is $118^\circ$, so the measure of the angle formed by Hawthorne and Emerson is $\frac{1}{2}(118^\circ) = 59^\circ$. The measure of the angle formed by Emerson and Thoreau is $134^\circ$, so the measure of the angle formed by Emerson and Dickinson is $\frac{1}{2}(134^\circ) = 67^\circ$. By the Angle Addition Postulate, the measure of the angle formed by Hawthorne and Dickinson is $59^\circ + 67^\circ = 126^\circ$. 

23. **Represent Real-World Problems** A carpenter is building a rectangular bookcase with diagonal braces across the back, as shown. The carpenter knows that $\angle ADC$ is a right angle and that $m\angle BDC$ is $32^\circ$ greater than $m\angle ADB$. Write and solve an equation to find $m\angle BDC$ and $m\angle ADB$.

\[
m\angle ADB + m\angle BDC = m\angle ADC
\]
\[
x + (x + 32) = 90
\]
\[
2x + 32 = 90
\]
\[
x = 29
\]
So, $m\angle ADB = 29^\circ$ and $m\angle BDC = 29 + 32 = 61^\circ$

24. **Describe the relationships among the four terms.**

The definitions of the terms “angle bisector” and “angle” are each built upon the definitions of the term below it. The definition of the term “ray” is built upon the undefined term “line” below it.

25. **Determine whether each of the following pairs of angles have equal measures.**

Select the correct answer for each lettered part.

A. $\angle KJL$ and $\angle IJM$
   - Yes
   - No
B. $\angle MJP$ and $\angle PJR$
   - Yes
   - No
C. $\angle JLP$ and $\angle NJR$
   - Yes
   - No
D. $\angle MJK$ and $\angle PIR$
   - Yes
   - No
E. $\angle KJR$ and $\angle MJR$
   - Yes
   - No

a. no; $m\angle LJM = 90^\circ - 42^\circ = 48^\circ \neq m\angle KJL$

b. yes; $m\angle NJP = 48^\circ$ so $m\angle MJP = 46^\circ + 48^\circ = 94^\circ$ and $m\angle PJR = 48^\circ + 46^\circ = 94^\circ$

c. yes; $m\angle NJP = 48^\circ$ and $m\angle LJM = 90^\circ - 42^\circ = 48^\circ$, so $m\angle LJP = 48^\circ + 46^\circ + 48^\circ = 142^\circ$ and $m\angle NJR = 48^\circ + 48^\circ + 46^\circ = 142^\circ$

d. no; $m\angle MJK = 90^\circ$, but $m\angle PJR = 48^\circ + 46^\circ = 94^\circ$

e. no; $m\angle KJR = 360^\circ - 90^\circ - 46^\circ - 48^\circ - 48^\circ - 46^\circ = 82^\circ$, but $m\angle MJP = 46^\circ + 48^\circ = 94^\circ$
26. Make a Conjecture  A rhombus is a quadrilateral with four sides of equal length. Use a compass and straightedge to bisect one of the angles in each of the rhombuses shown. Then use your results to state a conjecture.

Constructions may vary. Sample:

![Diagram of rhombuses with angle bisectors]

In a rhombus, the bisector of an angle also bisects the opposite angle.

H.O.T. Focus on Higher Order Thinking

27. What if? What happens if you perform the steps for constructing an angle bisector when the given angle is a straight angle? Does the construction still work? If so, explain why and show a sample construction. If not, explain why not.

Yes; the construction still works. In this case, the construction produces two right angles since each has half the measure of a straight angle (180°).

28. Critical Thinking Use a compass and straightedge to construct an angle whose measure is \( m\angle A - m\angle B \). Use a protractor to check your construction.

![Diagram of angles with angle bisectors]

29. Communicate Mathematical Ideas Explain the steps for using a compass and straightedge to construct an angle with \( \frac{1}{4} \) the measure of a given angle. Then draw an angle and show the construction.

**Construct the bisector of the given angle.**
**Then construct the bisector of one of the angles that was formed.**

![Diagram of angle bisectors]