Practice Integration

p. 471: 11-37 odd (skip 21, 25)

11. $\int_0^T (x^4 - 8x + 7)\,dx = \left[\frac{1}{5}x^5 - 4x^2 + 7x\right]_0^T = \left(\frac{1}{5}T^5 - 4T^2 + 7T\right) - 0 = \frac{1}{5}T^5 - 4T^2 + 7T$

13. Let $u = 1 - x$, so $du = -dx$ and $dx = -du$. When $x = 0, u = 1$; when $x = 1, u = 0$. Thus,

$$\int_0^1 (1-x)^9\,dx = \int_1^0 -u^9\,du = \int_0^1 u^9\,du = \frac{1}{10}u^{10}\Big]_0^1 = \frac{1}{10}(1-0) = \frac{1}{10}.$$

15. $\int_0^1 \left(\sqrt[4]{t} + 1\right)^2 dt = \int_0^1 \left(u^{1/2} + 2u^{1/4} + 1\right) dt = \left[\frac{2}{3}u^{3/2} + \frac{8}{5}u^{5/4} + u\right]_0^1 = \left(\frac{2}{3} + \frac{8}{5} + 1\right) - 0 = \frac{49}{15}$

17. Let $u = 1 + y^3$, so $du = 3y^2dy$ and $y^2dy = \frac{1}{3}du$. When $y = 0, u = 1$; when $y = 2, u = 9$. Thus,

$$\int_0^2 y^2\sqrt{1+y^3}\,dy = \int_1^9 u^{1/2}\left(\frac{1}{3}du\right) = \frac{1}{3}\left(\frac{2}{3}u^{3/2}\right)\Big]_1^9 = \frac{2}{9}(27-1) = \frac{52}{9}.$$

19. Let $u = 3\pi t$, so $du = 3\pi dt$. When $t = 0, u = 1$; when $t = 1, u = 3\pi$. Thus,

$$\int_0^1 \sin(3\pi t)\,dt = \int_0^{3\pi} \sin u\left(\frac{1}{3\pi}du\right) = \frac{1}{3\pi} - \cos u\Big]_0^{3\pi} = -\frac{1}{3\pi}(-1-1) = \frac{2}{3\pi}.$$

23. Let $u = e^x$, so $du = e^x dx$. When $x = 0, u = 1$; when $x = 1, u = e$. Thus,

$$\int_0^1 \frac{e^x}{1+e^{2x}}\,dx = \int_1^e \frac{1}{1+u^2}\,du = \arctan u\Big]_1^e = \arctan e - \arctan 1 = \arctan e - \frac{\pi}{4}.$$

27. Let $u = 1 + \cot x$. Then $du = -\csc^2 x\,dx$, so $\int \frac{\csc^2 x}{1+\cot x}\,dx = \int -\frac{1}{u}\,du = -\ln|u| + C = -\ln|1+\cot x| + C.$

29. Let $u = \cos x$. Then $du = -\sin x\,dx$, so

$$\int \sin x\cos(\cos x)\,dx = -\int \cos u\,du = -\sin u + C = -\sin(\cos x) + C.$$

31. Let $u = \ln x$. Then $du = \frac{1}{x}dx$, so $\int \frac{\sin(\ln x)}{x}\,dx = \int \sin u\,du = -\cos u + C = -\cos(\ln x) + C.$

33. Let $u = x^2$. Then $du = 2x\,dx$, so $\int \frac{x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1} u + C = \frac{1}{2}\sin^{-1}(x^2) + C.$

35. $\int \frac{1+x^4}{x^3}\,dx = \int \left(\frac{1}{x^3} + x\right) dx = \int \left(x^{-3} + x\right) dx = -\frac{1}{2}x^{-2} + \frac{1}{2}x^2 + C$

37. Let $u = 1 + \tan t$, so $du = \sec^2 t\,dt$. When $t = 0, u = 1$; when $t = \frac{\pi}{4}, u = 2$. Thus,

$$\int_0^{\pi/4} (1+\tan t)^3 \sec^2 t\,dt = \int_1^2 u^3\,du = \frac{1}{4}u^4\Big]_1^2 = \frac{1}{4}\left(2^4 - 1^4\right) = \frac{1}{4}(16-1) = \frac{15}{4}.$$

p. 601: 1, 3-4, 6, 9-10, 12, 25, 37

1. $\int_1^2 \frac{(x+1)^2}{x}\,dx = \int_1^2 \frac{x^2+2x+1}{x}\,dx = \int_1^2 \left(x + 2 + \frac{1}{x}\right) dx = \left(\frac{1}{2}x^2 + 2x + \ln|x|\right)\Big|_1^2$

$$= (2 + 4 + \ln 2) - \left(\frac{1}{2} + 2 + 0\right) = \frac{7}{2} + \ln 2$$

3. Using the substitution $u = \sin x$, $du = \cos x dx$,

$$\int \frac{e^{\sin x}}{\sec x} dx = \int \cos x e^{\sin x} dx = \int e^u du = e^u + C = e^{\sin x} + C$$

4. Use integration by parts with $u = t$, $du = dt$, $dv = \sin 2t$, $v = -\frac{1}{2}\cos 2t$:

$$\int_0^{\pi/6} t \sin 2t \, dt = \left(-\tfrac{1}{2}t\cos 2t\right)\Big|_0^{\pi/6} - \int_0^{\pi/6} -\tfrac{1}{2}\cos 2t \, dt = \left(-\tfrac{\pi}{12}\cdot\tfrac{1}{2}\right) - (0) + \left(\tfrac{1}{4}\sin 2t\right)\Big|_0^{\pi/6} = -\tfrac{\pi}{24} + \tfrac{1}{8}\sqrt{3}$$

6. Use parts with $u = \ln x$, $dv = x^5 \, dx$.

Then $\int_1^2 x^5 \ln x \, dx = \left(\tfrac{1}{6}x^6 \ln x\right)\Big|_2^3 - \int_1^2 \tfrac{1}{6}x^5 \, dx = \tfrac{64}{6}\ln 2 - 0 - \left(\tfrac{1}{36}x^6\right)\Big|_1^2 = \tfrac{32}{3}\ln 2 - \left(\tfrac{64}{36} - \tfrac{1}{36}\right) = \tfrac{32}{3}\ln 2 - \tfrac{7}{4}$

9. Let $u = \ln t, du = dt/t$. Then $\int \frac{\sin(\ln t)}{t} dt = \int \sin u \, du = -\cos u + C = -\cos(\ln t) + C$

10. Let $u = \tan^{-1} x$, $du = dx/(1+x^2)$. Then

$$\int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx = \int_0^{\pi/4} \sqrt{u} \, du = \frac{2}{3}\left(u^{3/2}\right)\Big|_0^{\pi/4} = \frac{2}{3}\left(\frac{\pi^{3/2}}{4^{3/2}} - 0\right) = \frac{2}{3}\cdot\frac{1}{8}\pi^{3/2} = \frac{1}{12}\pi^{3/2}.$$

12. Let $u = e^{2x}$, $du = 2e^{2x} \, dx$. Then

$$\int \frac{e^{2x}}{1+e^{4x}} dx = \int \frac{1}{1+u^2} \cdot \tfrac{1}{2} du = \tfrac{1}{2}\tan^{-1} u + C = \tfrac{1}{2}\tan^{-1} e^{2x} + C$$

25. $\int x \sin x \cos x \, dx = \int \tfrac{1}{2} x \sin 2x \, dx$

Let $u = \frac{1}{2}x$, $dv = \sin 2x \, dx$. Then $du = \frac{1}{2}dx$, $v = -\frac{1}{2}\cos 2x$. Therefore,

$$\int x \sin x \cos x \, dx = \int \tfrac{1}{2} x \sin 2x \, dx = -\tfrac{1}{4}x\cos 2x + \int \tfrac{1}{4}\cos 2x \, dx = -\tfrac{1}{4}x\cos x + \tfrac{1}{8}\sin 2x + C$$

37. Let $u = \sqrt{x}$, $du = 1/(2\sqrt{x}) \, dx$. Then $\int \frac{2^{\sqrt{x}}}{\sqrt{x}} dx = \int 2^u (2du) = 2\cdot\frac{2^u}{\ln 2} + C = \frac{2^{\sqrt{x}+1}}{\ln 2} + C$.