p. 697: 11-27 odd, 40-44

11. $\frac{2}{3}-\frac{2}{5}+\frac{2}{7}-\frac{2}{9}+\frac{2}{11}-\cdots=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{2n+1}$. Now $b_n=\frac{2}{2n+1}>0$, $\{b_n\}$ is decreasing, and $\lim_{n\to\infty} b_n=0$, so the series converges by the Alternating Series Test.

13. $\frac{1}{\ln 3}-\frac{1}{\ln 4}+\frac{1}{\ln 5}-\frac{1}{\ln 5}+\frac{1}{\ln 7}-\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\ln(n+2)}$. Now $b_n=\frac{1}{\ln(n+2)}>0$, $\{b_n\}$ is decreasing, and $\lim_{n\to\infty} b_n=0$, so the series converges by the Alternating Series Test.

15. $\sum_{n=0}^{\infty}(-1)^{n+1}b_n=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n+1}}$. Now $b_n=\frac{1}{\sqrt{n+1}}>0$, $\{b_n\}$ is decreasing, and $\lim_{n\to\infty} b_n=0$, so the series converges by the Alternating Series Test.

17. $\sum_{n=1}^{\infty}(-1)^n b_n=\sum_{n=1}^{\infty}(-1)^n\frac{n^2}{n^2+n+1}$. Now $\lim_{n\to\infty} b_n=\lim_{n\to\infty}\frac{n^2}{n^2+n+1}=\lim_{n\to\infty}\frac{1}{1+1/n+1/n^2}=1\neq 0$. Because $\lim_{n\to\infty} b_n\neq 0$ (in fact the limit does not exist), the series diverges by the Test for Divergence.

19. $b_n=\frac{\sqrt{n}}{2n+3}>0$ for $n\geq 1$. $\{b_n\}$ is decreasing for $n\geq 2$ since

$$\left(\frac{\sqrt{x}}{2x+3}\right)'=\frac{(2x+3)\left(\frac{1}{2}x^{-1/2}\right)-x^{1/2}(2)}{(2x+3)^2}=\frac{\frac{1}{2}x^{-1/2}\left[(2x+3)-4x\right]}{(2x+3)^2}=\frac{3-2x}{2\sqrt{x}(2x+3)^2}<0 \text{ for } x>\tfrac{3}{2}.$$

In addition, $\lim_{n\to\infty} b_n=\lim_{n\to\infty}\frac{\sqrt{n}/\sqrt{n}}{(2n+3)/\sqrt{n}}=\lim_{n\to\infty}\frac{1}{2\sqrt{n}+3\sqrt{n}}=0$. Thus, the series $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{2n+3}$ converges by the Alternating Series Test.

21. $b_n=ne^{-n}=\frac{n}{e^n}>0$ for $n\geq 1$. $\{b_n\}$ is decreasing for $n\geq 1$ since $\left(xe^{-x}\right)'=x(-e^{-x})+e^{-x}=e^{-x}(1-x)<0$ for $x>1$. In addition, $\lim_{n\to\infty} b_n=0$ since by l'Hopital's Rule $\lim_{x\to\infty}\frac{x}{e^x}=\lim_{x\to\infty}\frac{1}{e^x}=0$. Thus, the series $\sum_{n=1}^{\infty}(-1)^{n+1}ne^{-n}$ converges by the Alternating Series Test.

23. $\lim_{n\to\infty} b_n=\lim_{n\to\infty}\arctan n=\frac{\pi}{2}$, so $\lim_{n\to\infty}(-1)^{n-1}\arctan n$ does not exist. Thus, the series $\sum_{n=1}^{\infty}(-1)^{n-1}\arctan n$ diverges by the Test for Divergence.

25. $\frac{n\cos n\pi}{2^n}=(-1)^n\frac{n}{2^n}\Rightarrow b_n=\frac{n}{2^n}>0$. $\{b_n\}$ is decreasing for $n\geq 2$ since $\left(x2^{-x}\right)'=x(-2^{-x}\ln 2)+2^{-x}$ $=2^{-x}(1-x\ln 2)<0$ for $x>\frac{1}{\ln 2}$. In addition, by l'Hopital's Rule, $\lim_{n\to\infty} b_n=\lim_{x\to\infty}\frac{x}{2^x}=\lim_{x\to\infty}\frac{1}{2^x\ln 2}=0$. Thus, the series $\sum_{n=0}^{\infty}\frac{n\cos n\pi}{2^n}$ converges by the Alternating Series Test.

27. $\lim_{n\to\infty}\cos\left(\frac{\pi}{n}\right)=\cos(0)=1$, so $\lim_{n\to\infty}(-1)^n\cos\left(\frac{\pi}{n}\right)$ does not exist and the series $\sum_{n=1}^{\infty}(-1)^n\cos\left(\frac{\pi}{n}\right)$ diverges by the Test for Divergence.

40. $\sum \frac{\sqrt{n}-3}{n^2}$ converges by comparison to $\sum \frac{1}{n^{3/2}}$ (convergent by p-series Test), so $\sum (-1)^n \frac{\sqrt{n}-3}{n^2}$ converges by Absolute Convergence Test. Therefore, choice (B) is correct.

41. If $p=2$, the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2+2}{n^p+4} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2+2}{n^2+4}$ does not converge by the Test for Divergence because $\lim_{n\to\infty} \frac{n^2+2}{n^2+4} = \lim_{n\to\infty} \frac{1+2/n^2}{1+4/n^2} = 1 \neq 0.$. However, if $p=3$, the terms $b_n = \frac{n^2+2}{n^3+4}$ are positive, and decreasing and $\lim_{n\to\infty} \frac{1/n + 2/n^3}{1+4/n^3} = 0$. Thus, the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2+2}{n^p+4} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2+2}{n^3+4}$ converges. Therefore, choice (**B**) is correct.

42. $|R_4| = |s - s_4| = |s_5| = \left| -\frac{4}{3^5} \right| = \frac{4}{243}$, option (**A**).

43. The error bound is $|R_n| = |s - s_n| = |s_{n+1}| = \frac{1}{(n+1)^2}$, so we need $\frac{1}{(n+1)^2} < \frac{1}{1000} \Rightarrow (n+1)^2 > 1000 \Rightarrow$ $n+1 > 10 \Rightarrow n > 11$. Of the values given, the smallest that is larger than 11 is (**B**), $n=24$.

44. $\lim_{n\to\infty} \sqrt{\frac{n}{n+1}} = \lim_{n\to\infty} \sqrt{\frac{1}{1+1/n}} = 1 \neq 0$, so series (**D**) diverges by the Test for divergence.

p. 706: 11-15 odd

11. $b_n = \frac{1}{\sqrt{n}} > 0$ for $n \geq 1$, $\{b_n\}$ is decreasing for $n \geq 1$, and $\lim_{n\to\infty} b_n = 0$, so $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ converges by the Alternating Series Test. To determine absolute convergence, note that $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges because it is a p-series with $p = \frac{1}{2} \leq 1$. Thus, the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ is conditionally convergent.

13. $0 < \frac{1}{n^3+1} < \frac{1}{n^3}$ for $n \geq 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is a convergent p-series ($p = 3 > 1$), so $\sum_{n=1}^{\infty} \frac{1}{n^3+1}$ converges by comparison and the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$ is absolutely convergent. 13

15. $b_n = \frac{n}{n^2+4} > 0$ for $n \geq 1$, $\{b_n\}$ is decreasing for $n \geq 2$, and $\lim_{n\to\infty} b_n = 0$, so $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+4}$ converges by the Alternating Series Test. To determine absolute convergence, choose $a_n = \frac{1}{n}$ to get

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1/n}{n/(n^2+4)} = \lim_{n\to\infty} \frac{n^2+4}{n^2} = \lim_{n\to\infty} \frac{1+4/n^2}{1} = 1 > 0,$$

so $\sum_{n=1}^{\infty} \frac{n}{n^2+4}$ diverges by the Limit Comparison Test with the harmonic series. Thus, the series $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+4}$ is conditionally convergent.