5.1

p. 392: 5-51 odd, 52-56, 60-63, 67-71 odd, 74

5. $f(x)=4x+7=4x^1+7 \Rightarrow F(x)=4\dfrac{x^{1+1}}{1+1}+7x+C=2x^2+7x+C$

7. $f(x)=2x^3-\frac{2}{3}x^2+5x \Rightarrow F(x)=2\dfrac{x^4}{4}-\dfrac{2}{3}\dfrac{x^3}{3}+5\dfrac{x^2}{2}+C=\frac{1}{2}x^4-\frac{2}{9}x^3+\frac{5}{2}x^2+C$

9. $f(x)=x(12x+8)=12x^2+8x \Rightarrow F(x)=12\cdot\frac{1}{3}x^3+8\cdot\frac{1}{2}x^2+C=4x^3+4x^2+C$

11. $f(x)=7x^{2/5}+8x^{-4/5} \Rightarrow F(x)=7\cdot\frac{5}{7}x^{7/5}+8\cdot 5x^{1/5}+C=5x^{7/5}+40x^{1/5}+C$

13. $f(x)=\sqrt{2}$ is a constant function, so $F(x)=\sqrt{2}x+C$.

15. $f(x)=3\sqrt{x}-2\sqrt[3]{x}=3x^{1/2}-2x^{1/3} \Rightarrow F(x)=3\left(\frac{2}{3}x^{3/2}\right)-2\left(\frac{3}{4}x^{4/3}\right)+C=2x^{3/2}-\frac{3}{2}x^{4/3}+C$

17. $f(x)=\dfrac{1}{5}-\dfrac{2}{x}=\dfrac{1}{5}-2\left(\dfrac{1}{x}\right)$ has domain $(-\infty,0)\cup(0,\infty)$, so $F(x)=\begin{cases}\frac{1}{5}x-2\ln|x|+C_1 & \text{if } x<0\\ \frac{1}{5}x-2\ln|x|+C_2 & \text{if } x>0\end{cases}$

19. $g(t)=\dfrac{1+t+t^2}{\sqrt{t}}=t^{-1/2}+t^{1/2}+t^{3/2} \Rightarrow G(t)=2t^{1/2}+\frac{2}{3}t^{3/2}+\frac{2}{5}t^{5/2}+C$

21. $h(\theta)=2\sin\theta-\sec^2\theta \Rightarrow H(\theta)=-2\cos\theta-\tan\theta+C_n$ on the interval $\left(n\pi-\frac{\pi}{2},n\pi+\frac{\pi}{2}\right)$.

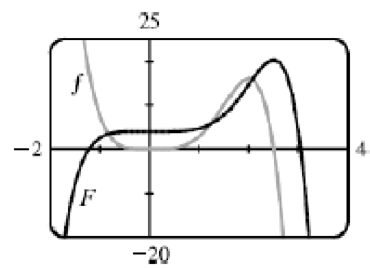
23. $f(x)=2^x+x^2 \Rightarrow$

$$F(x)=\frac{2^x}{\ln 2}+\frac{x^3}{3}+C$$

25. $f(x)=\dfrac{2x^4+4x^3-x}{x^3},\quad x>0;\ f(x)=2x+4-x^{-2} \Rightarrow$

$F(x)=2\left(\frac{1}{2}x^2\right)+4x-\dfrac{x^{-2+1}}{-2+1}+C=x^2+4x+\dfrac{1}{x}+C,\ x>0$

27. $f(x)=5x^4-2x^5 \Rightarrow F(x)=5\cdot\frac{1}{5}x^5-2\cdot\frac{1}{6}x^6+C=x^5-\frac{1}{3}x^6+C$.
$F(0)=4 \Rightarrow 0^5-\frac{1}{3}\cdot 0^6+C=4 \Rightarrow C=4$. So, $F(x)=x^5-\frac{1}{3}x^6+4$. The graph confirms our answer since $f(x)=0$ when F has a local maximum, f is positive when F is increasing, and f is negative when F is decreasing.



29. $f''(x)=20x^3-12x^2+6x \Rightarrow f'(x)=20\left(\dfrac{x^4}{4}\right)-12\left(\dfrac{x^3}{3}\right)+6\left(\dfrac{x^2}{2}\right)+C=5x^4-4x^3+3x^2+C \Rightarrow$

$f(x)=5\left(\dfrac{x^5}{5}\right)-4\left(\dfrac{x^4}{4}\right)+3\left(\dfrac{x^3}{3}\right)+Cx+D=x^5-x^4+x^3+Cx+D$

31. $f''(x)=2x+3e^x \Rightarrow f'(x)=x^2+3e^x+C \Rightarrow \frac{1}{3}x^3+3e^x+Cx+D$

33. $f'''(t)=12+\sin t \Rightarrow f''(t)=12t-\cos t+C_1 \Rightarrow f'(t)=6t^2-\sin t+C_1t+D \Rightarrow$
$f(t)=2t^3+\cos t+Ct^2+Dt+E$, where $C=\frac{1}{2}C_1$.

35. $f'(x)=1+3\sqrt{x} \quad \Rightarrow \quad f(x)=x+3\left(\frac{2}{3}x^{3/2}\right)+C=x+2x^{3/2}+C \quad f(4)=2(8)+C=25 \Rightarrow$
$20+C=25 \Rightarrow C=5$, so $f(x)=x+2x^{3/2}+5$.

37. $f'(t)=\dfrac{4}{1+t^2} \Rightarrow f(t)=4\arctan t+C.\quad f(1)=4\left(\dfrac{\pi}{4}\right)+C=0 \Rightarrow \pi+C=0 \Rightarrow C=-\pi,$
so $f(t)=4\arctan t-\pi.$

39. $f'(x)=5x^{2/3} \Rightarrow f(x)=5\left(\frac{3}{5}x^{5/3}\right)+C=3x^{5/3}+C.\ f(8)=3\cdot 32+C=21 \Rightarrow 96+C=21 \Rightarrow =-75,$ so
$f(x)=3x^{5/3}-75.$

41. $f'(t)=\sec t(\sec t+\tan t)=\sec^2 t+\sec t\tan t,\ -\frac{\pi}{2}<t<\frac{\pi}{2} \Rightarrow f(t)=\tan t+\sec t+C.$
$f\left(\frac{\pi}{4}\right)=1+\sqrt{2}+C=-1 \Rightarrow C=-2-\sqrt{2},$ so $f(t)=\tan t+\sec t+-2-\sqrt{2}.$
Note: The fact that f is defined and continuous on $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ means that we have only one constant of integration.

43. $f''(x)=-2+12x-12x^2 \Rightarrow f'(x)=-2x+6x^2-4x^3+C.\ f'(0)=C$ and $f'(0)=12 \Rightarrow C=12,$ so
$f'(x)=-2x+6x^2-4x^3+12,$ and hence, $f(x)=-x^2+2x^3-x^4+12x+D.\ f(0)=D=4,$ so
$f(x)=-x^2+2x^3-x^4+12x+4.$

45. $f''(\theta)=\sin\theta+\cos\theta \Rightarrow f'(\theta)=-\cos\theta+\sin\theta+C.\ f'(0)=-1+C=4 \Rightarrow C=5,$ so
$f'(\theta)=-\cos\theta+\sin\theta+5.$ Therefore, $f(\theta)=-\sin\theta-\cos\theta+5\theta+D.\ f(0)=-1+D=3 \Rightarrow D=4,$ so
$f(\theta)=-\sin\theta-\cos\theta+5\theta+4.$

47. $f''(x)=4+6x+24x^2 \Rightarrow f'(x)=4x+3x^2+8x^3+C \Rightarrow f(x)=2x^2+x^3+24x^4+Cx+D.$
$f(0)=D=3,$ so $f(x)=2x^2+x^3+2x^4+Cx+3.\ f(1)=8+C=10 \Rightarrow C=2,$ so
$f(x)=2x^2+x^3+24x^4+2x+3.$

49. $f''(t)=\sqrt[3]{t}-\cos t=t^{1/3}-\cos t \Rightarrow f'(t)=\frac{3}{4}t^{4/3}-\sin t+C \Rightarrow f(t)=\frac{9}{28}t^{7/3}+\cos t+Ct+D.$
$f(0)=0+1+0+D=2 \Rightarrow D=1,$ so $f(t)=\frac{9}{28}t^{7/3}+\cos t+Ct+1.\ f(1)=\frac{9}{28}+\cos 1+C+1=2 \Rightarrow$ so
$C=2-\frac{9}{28}-\cos 1-1=\frac{19}{28}-\cos 1,$ so $\frac{9}{28}t^{7/3}+\cos t+(\frac{19}{28}-\cos 1)t+1.$

51. $f'''(x)=\cos x \Rightarrow f''(x)=\sin x+C.\ f''(0)=C=3.\ f''(x)=\sin x+3 \Rightarrow f'(x)=-\cos x+3x+D.$
$f'(0)=-1+D=2 \Rightarrow D=3.\ f'(x)=-\cos x+3x+3 \Rightarrow f(x)=-\sin x+\frac{3}{2}x^2+3x+E.$
$f(0)=E=1 \Rightarrow\ f(x)=-\sin x+\frac{3}{2}x^2+3x+1.$

52. $f'(x)=\cos x \Rightarrow f(x)=\sin x+C,$ and $f\left(\frac{\pi}{2}\right)=0=\sin\left(\frac{\pi}{2}\right)+C \Rightarrow 0=1+C \Rightarrow C=-1,$ so
$f(x)=\sin x-1,$ which is option (**B**).

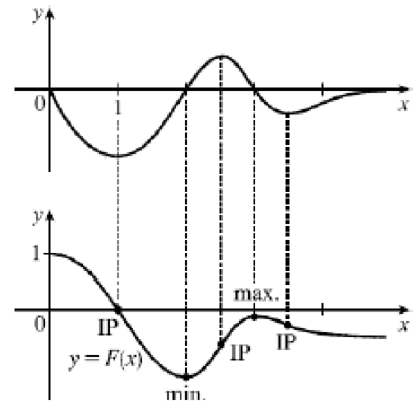
53. $f'(x)=2x-x^{-1} \Rightarrow f(x)=x^2-\ln|x|+C,$ and $f(-1)=1=1+0+C \Rightarrow C=0,$ so $f(x)=x^2-\ln|x|,$
which is option (**D**).

54. $f''(x)=6x \Rightarrow f'(x)=3x^2+C \Rightarrow f(x)=x^3+Cx+D.$ Therefore, (**C**) $2x^3+\dfrac{x}{2}$ cannot be $f(x).$

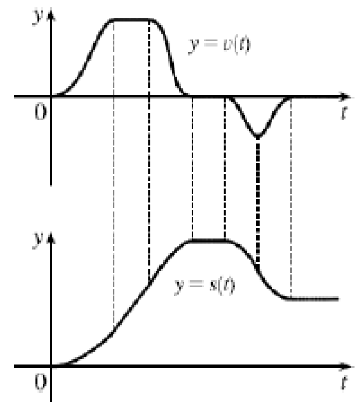
55. $f(x)=\dfrac{1}{x^2}=x^{-2} \Rightarrow F(x)=-\dfrac{1}{x}+C,$ and $F(1)=2=-1+C \Rightarrow C=3 \Rightarrow F(x)=-\dfrac{1}{x}+3,$ (**B**).

56. "The slope of its tangent line at $(x,f(x))$ is 3–4x" means that $f'(x)=3-4x,$ so $f(x)=3x-2x^2+C.$
"The graph of f passes through the point (2, 5)" means that $f(2)=5,$ but $f(2)=3(2)-2(2)^2+C,$ so
$5=6-8+C \Rightarrow C=7.$ Thus, $f(x)=3x-2x^2+7$ and $f(1)=3-2+7=8.$

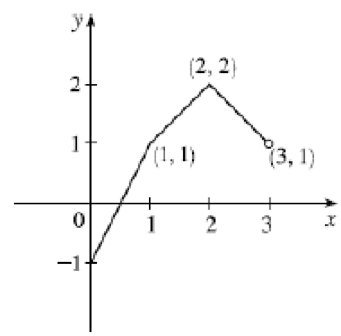
60. The graph of F must start at $(0,1)$. Where the given graph, has a local minimum or maximum, the graph of F will have an inflection point. Where f is negative (or positive), F is decreasing (or increasing). Where f changes from negative to positive, F will have a minimum. Where f changes from positive to negative, F will have a maximum. Where f is decreasing (increasing), F is concave up (down).



61. Where v is positive (or negative), s is increasing (or decreasing). Where v is increasing (decreasing), s is concave up (down). Where v is horizontal (a steady velocity), s is linear.



62. $f'(x)=\begin{cases}2 & \text{if } 0\le x<1\\ 1 & \text{if } 1<x<2\\ -1 & \text{if } 2<x<3\end{cases} \Rightarrow f(x)=\begin{cases}2x+C & \text{if } 0\le x<1\\ x+D & \text{if } 1<x<2\\ -x+E & \text{if } 2<x<3\end{cases}$



$f(0)=-1 \Rightarrow 2(0)+C=-1 \Rightarrow C=-1$. Starting at the point $(0,-1)$ and moving to the right on a line with slope 2 gets us to the point $(1,1)$. The slope for $1<x<2$ is 1, so we get to the point $(2,2)$. Here we have used the fact that f is continuous. We can include the point on either the first or second part of f. The line connecting $(1,1)$ to $(2,2)$ is $y=x$, so $D=0$. The slope for $2<x<3$ is -1, so we get to $(3,1)$.

$f(2)=2 \Rightarrow -2+E=2 \Rightarrow E=4$. Thus, $f(x)=\begin{cases}2x-1 & \text{if } 0\le x<1\\ x & \text{if } 1<x<2\\ -x+4 & \text{if } 2<x<3\end{cases}$. Note that $f'(x)$ does not exist at $x=1,2$ or 3.

63. The given graph is positive on $(0,1.5)$ and $(4,\infty)$, so f must be increasing on these intervals. The graph of f' is negative on $(1,5,4)$ so f must be decreasing on this interval. $f'=0$ when $x=1.5$, and 4 so f must be horizontal at $x=1.5$, and $x=4$. Finally, f' is increasing on $(3,\infty)$ so f must be concave up on this interval. Thus only graph **(B)** could be the graph of f.

67. $v(t)=s'(t)=\sin t-\cos t \Rightarrow s(t)=-\cos t-\sin t+C$. $s(0)=-1+C$ and $s(0)=0 \Rightarrow C=1$, so $s(t)=-\cos t-\sin t+1$.

69. $a(t) = v'(t) = 2t + 1 \Rightarrow v(t) = t^2 + t + C.$ $v(0) = C$ and $v(0) = -2 \Rightarrow C = -2,$ so $v(t) = t^2 + t - 2$ and $s(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2 - 2t + D.$ $s(0) = D$ and $s(0) = 3 \Rightarrow D = 3,$ so $(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2 - 2t + 3.$

71. $a(t) = v'(t) = 10\sin t + 3\cos t \Rightarrow v(t) = -10\cos t + 3\sin t + C \Rightarrow s(t) = -10\sin t - 3\cos t + Ct + D.$ $s(0) = -3 + D$ and $s(2\pi) = -3 + 2\pi C + D = 12 \Rightarrow D = 3$ and $C = \frac{6}{\pi}.$

Thus, $s(t) = -10\sin t - 3\cos t + \frac{6}{\pi}t + 3.$

74. (a) $r'(t) = \dfrac{1}{t^2} + \dfrac{t^2}{4} = t^{-2} + \frac{1}{4}t^2 \Rightarrow r(t) = -t^{-1} + \frac{1}{12}t^3 + C.$ $r(t) = -t^{-1} + \frac{1}{12}t^3 + \frac{47}{12}.$

$r(2) = -\frac{1}{2} + \frac{1}{12} \cdot 2^3 + \frac{47}{12} = \frac{49}{12} = 4.083$ barrels per hour.

(b) $A(t) = -\ln|t| + \dfrac{1}{48}t^4 + \frac{47}{12}t + D.$ $A(1) = 4 = -\ln 1 + \dfrac{1}{48} + \frac{47}{12} + D \Rightarrow D = \frac{1}{16} \Rightarrow$

$A(t) = -\ln|t| + \dfrac{1}{48}t^4 + \frac{47}{12}t + \frac{1}{16}.$

(c) At time $t = 2$, $r'(t) = \frac{1}{2^2} + \frac{2^2}{4} > 0$, so the rate of leakage is increasing