Calculus BC Semester 1 Sample Final

Section I

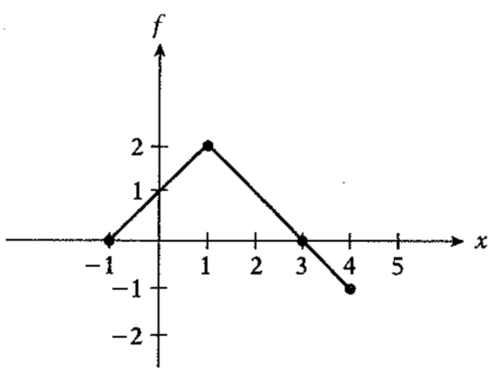
Part A

No calculator is allowed for these questions.

1. $f(x) = 2x^3 - 6x^2 + 6x - 1$ has a point of inflection located at

(A) $(0, -1)$
(B) $(1, 1)$
(C) $(2, 3)$
(D) $(1, 0)$
(E) $(-1, 1)$

2. Find the average value of $f(x)$ on the interval $[-1, 4]$ in the figure shown.



(A) $-\frac{1}{5}$
(B) $\frac{7}{10}$
(C) $\frac{9}{10}$
(D) $\frac{7}{2}$
(E) $\frac{35}{2}$

3. $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} 2\sin(x^2)\,dx =$

(A) -1
(B) $-\frac{\sqrt{2}}{2}$
(C) $-\frac{1}{2}$
(D) 0
(E) $\frac{1}{2}$

4. A function $f(x)$ is continuous on the closed interval $[a, b]$. Which of the following must be true?

(A) $f(x)$ has a maximum on $[a, b]$.
(B) $f(x)$ has a point of inflection on $[a, b]$.
(C) $f'(c) = \frac{f(b) - f(a)}{b - a}$ for at least one c in the interval $[a, b]$.
(D) $f'(c) = 0$ for at least one c in the interval $[a, b]$.
(E) $f(x)$ has a critical value on the interval $[a, b]$.

5. Find the slope of the tangent to the graph of $3xy - 2x + 3y^2 = 5$ at the point $(2, 1)$.

(A) $-\frac{1}{3}$
(B) $-\frac{1}{12}$
(C) $\frac{1}{12}$
(D) 1
(E) undefined

6. If $f(x) = \left(g(x)\right)^5$, $g(2) = -1$, and $f'(2) = 5$, find $g'(2)$.

(A) -5
(B) 0
(C) $\frac{1}{5}$
(D) 1
(E) 5

7. $\frac{d}{dx}\left(\frac{x+1}{x+2}\right) =$

(A) 0
(B) 1
(C) $\frac{1}{x+2}$
(D) $\frac{1}{(x+2)^2}$
(E) $-\frac{1}{(x+2)^2}$

8. What is the instantaneous rate of change of $f(x) = \ln(\tan^2 x)$ at $x = \frac{\pi}{4}$?

(A) 0
(B) 1
(C) $\frac{\sqrt{3}}{2}$
(D) 4
(E) undefined

9. Consider the function

$$f(x) = \begin{cases} 2x^2 - x^3, & x < 2 \\ e^{2x-4}, & x \geq 2 \end{cases}.$$

Find $\lim_{x \to 2} f(x)$.

(A) 0
(B) 1
(C) 2
(D) 8
(E) does not exist

10. $f(x) = e^{\sin^2 x}$. $f'(x) =$

(A) $e^{\sin^2 x}$
(B) $2 \sin x e^{\sin^2 x}$
(C) $2 \sin x \cos x e^{\sin^2 x}$
(D) $e^{2 \cos x}$
(E) $e^{\cos^2 x}$

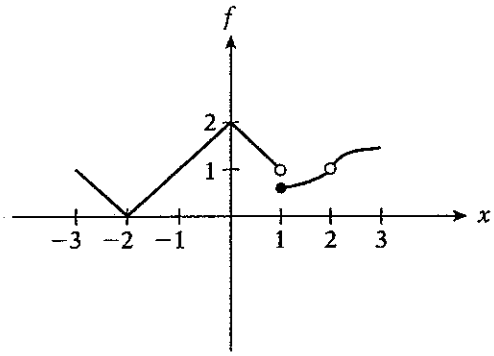
11. $\int_0^{\frac{\pi}{3}} \sec x \tan x \, dx =$

(A) 0
(B) 1
(C) $\sqrt{2} - 1$
(D) $\sqrt{3} - 1$
(E) undefined

12. The position function for a particle's motion on a line is $x(t) = t^3 - t^2 - t + 1, t \geq 0$. At what value(s) of t is the particle at rest?

(A) $t = 0$
(B) $t = 1$
(C) $t = -1$ and $t = 1$
(D) $t = \frac{1}{3}$
(E) no value of t

13. At which value(s) of x on $[-3, 3]$ is $f(x)$ discontinuous?



(A) $x = -2$
(B) $x = -2$ and $x = 1$
(C) $x = 1$
(D) $x = 1$ and $x = 2$
(E) $x = -2, x = 1$, and $x = 2$

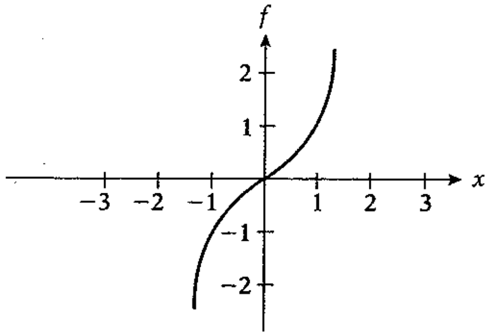
14. $\int_0^1 \frac{x^2 - 1}{x^2 + 1} dx =$

(A) -1
(B) $1 - \frac{\pi}{2}$
(C) $1 - \frac{\pi}{3}$
(D) $1 - \frac{\pi}{4}$
(E) 1

15. $\frac{d}{dx}\left(\int_2^{2x} \sqrt[3]{1+t}\, dt\right) =$

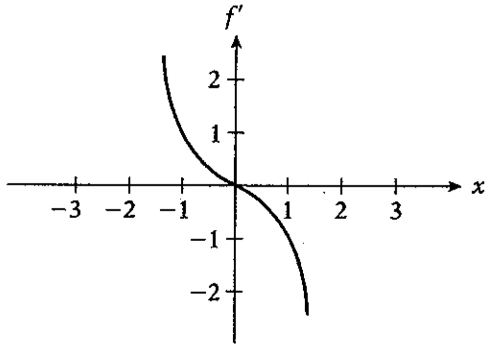
(A) $\sqrt[3]{1+2x} - \sqrt[3]{3}$
(B) $2\sqrt[3]{1+2x} - \sqrt[3]{3}$
(C) $2\sqrt[3]{1+2x}$
(D) $\sqrt[3]{1+4x^2}$
(E) $\sqrt[3]{1+2x}$

16. Consider the graph of f shown.

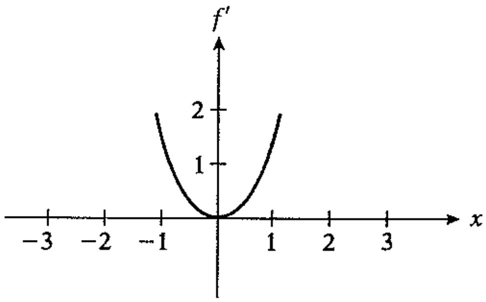


Which of the following is the graph of f'?

(A)



(B)



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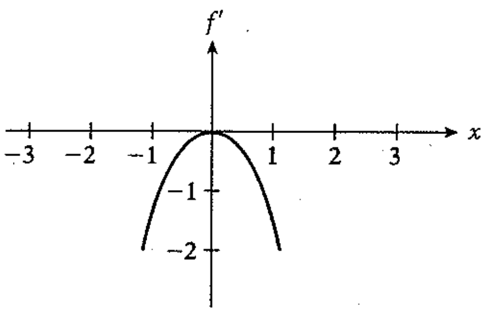

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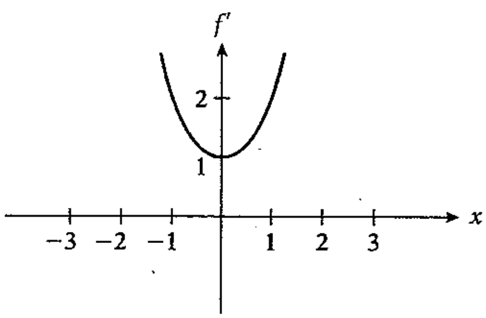
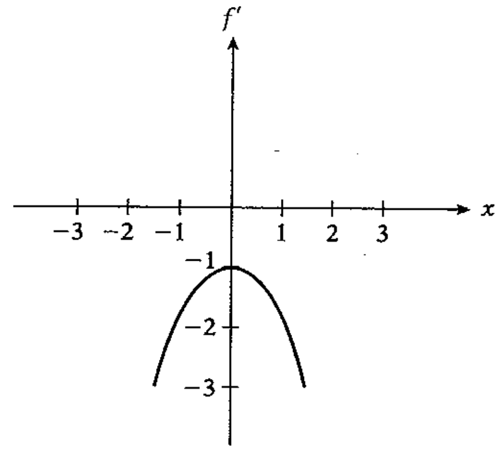


(E)



17. Find the equation of the line tangent to $y = \tan^2 x$ at $x = \frac{\pi}{3}$.

(A) $y - 3 = 8\sqrt{3}\left(x - \frac{\pi}{3}\right)$

(B) $y - 3 = 4\sqrt{3}\left(x - \frac{\pi}{3}\right)$

(C) $y - 3 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$

(D) $y - 1 = 4\left(x - \frac{\pi}{3}\right)$

(E) $y - 1 = 4\sqrt{3}\left(x - \frac{\pi}{3}\right)$

18. $f'(x) = 2x(x + 1)(x + 2)^2$. $f(x)$ has

(A) a relative maximum at $x = -1$ and a relative minimum at $x = 0$

(B) relative maxima at $x = -1$ and $x = -2$, and a relative minimum at $x = 0$

(C) a relative maximum at $x = -2$ and a relative minimum at $x = 0$

(D) only a relative minimum at $x = 0$

(E) only a relative maximum at $x = -2$

19. The area under the graph of $y = \frac{1}{x}$ from $x = 1$ to $x = a$ (where $a > 1$) is equal to the area under the curve from $x = p$ (where $p < 1$) to $x = 1$. Express p in terms of a.

(A) $p = \frac{1}{a}$

(B) $p = -a$

(C) $p = \frac{1}{a^2}$

(D) $p = \frac{1}{\ln a}$

(E) $p = -\ln a$

20. $\frac{dy}{dt} = ky$ and $y(1) = 1$. Find $y(t)$ in terms of k and t.

(A) $y = kt^2$

(B) $y = k\sqrt{t}$

(C) $y = e^{-kt}$

(D) $y = e^{k(t-1)}$

(E) $y = e^{kt} + 1$

21. Consider the function $f(x) = x^4 + 2x^2 + 1$. On what interval is $f(x)$ increasing?

(A) $(-\infty, \infty)$

(B) $(-\infty, 0)$

(C) $(0, \infty)$

(D) $(1, \infty)$

(E) $(-\infty, 1)$

22. $a(t) = 6t - 12$ for $0 \leq t \leq 4$. If $v(0) = 18$, find the maximum velocity on the interval $[0, 4]$.

(A) 0

(B) 2

(C) 4

(D) 6

(E) 18

23. $f(x) = \frac{1}{2}(2x + 5)^3$. $f'(x) =$

(A) $\frac{3}{2}(2x + 5)^2$

(B) $3(2x + 5)^2$

(C) $3(2x + 5)$

(D) $\frac{3}{2}(2x + 5)$

(E) $6(2x + 5)$

24. The area enclosed by the curves $y = x^2 - 4$ and $y = 2x - 4$ can be represented by the integral

(A) $\int_{-4}^{0}(x^2 - 2x)\,dx$

(B) $\int_{0}^{2}(2x - x^2)\,dx$

(C) $\int_{-4}^{0}(2x - x^2)\,dx$

(D) $\int_{0}^{2}(x^2 - 2x)\,dx$

(E) $\int_{-4}^{2}(x^2 - 2x)\,dx$

25. $\lim\limits_{x \to -\infty} \frac{2x}{\sqrt{x^2 + 1}} =$

(A) -2

(B) -1

(C) 1

(D) 2

(E) undefined

26. $g(x)$ is a differentiable function on the closed interval $[a, b]$. Which of the following must be true?

(A) For every x in $[a, b]$, $g(x)$ is between $g(a)$ and $g(b)$.

(B) For every k between $g(a)$ and $g(b)$, there is a value c in $[a, b]$ such that $g(c) = k$.

(C) There is at least one x in $[a, b]$ such that $g'(x) = 0$.

(D) $\lim\limits_{x \to \infty} g(x)$ does not exist.

(E) $g''(0) = 0$

27. The region enclosed by the graphs of $y = x^{2/3}$, $y = 4$, and the y-axis is rotated about the line $y = 4$. The volume of the solid generated can be represented by the integral

(A) $2\pi \int_{0}^{8}(4 - x^{2/3})^2\,dx$

(B) $\pi \int_{0}^{8}(4 - x^{2/3})^2\,dx$

(C) $2\pi \int_{0}^{4}(4 - x^{2/3})^2\,dx$

(D) $\pi \int_{0}^{4}(16 - x^{4/3})\,dx$

(E) $\pi \int_{0}^{8}(16 - x^{4/3})\,dx$

28. On what interval(s) is the graph of $f(x) = \frac{x}{x^2 + 1}$ concave down?

(A) $\left(0, \sqrt{3}\right)$

(B) $\left(-\sqrt{3}, 0\right)$

(C) $(0, \infty) \cup \left(-\sqrt{3}, 0\right)$

(D) $\left(-\infty, -\sqrt{3}\right) \cup \left(0, \sqrt{3}\right)$

(E) $\left(\sqrt{3}, \infty\right)$

Section I

Part B

A graphing calculator is required for some questions.

1. Find $\frac{dy}{dx}$ when $y = 0$ if $x \cos y - \sin x - 2 = 0$.

(A) 0
(B) 0.637
(C) 1
(D) 2.554
(E) undefined

2. If $v(t) = \ln(t^2 + t + 1)$, then $a(1) =$

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) 1
(D) $\frac{4}{3}$
(E) 3

3. $y = \sec^2(2x + \pi)$. Find $y'\left(\frac{\pi}{2}\right)$.

(A) 0
(B) 2
(C) 4
(D) 8
(E) undefined

4. $y = \frac{e^{2x-1}}{x}$ has

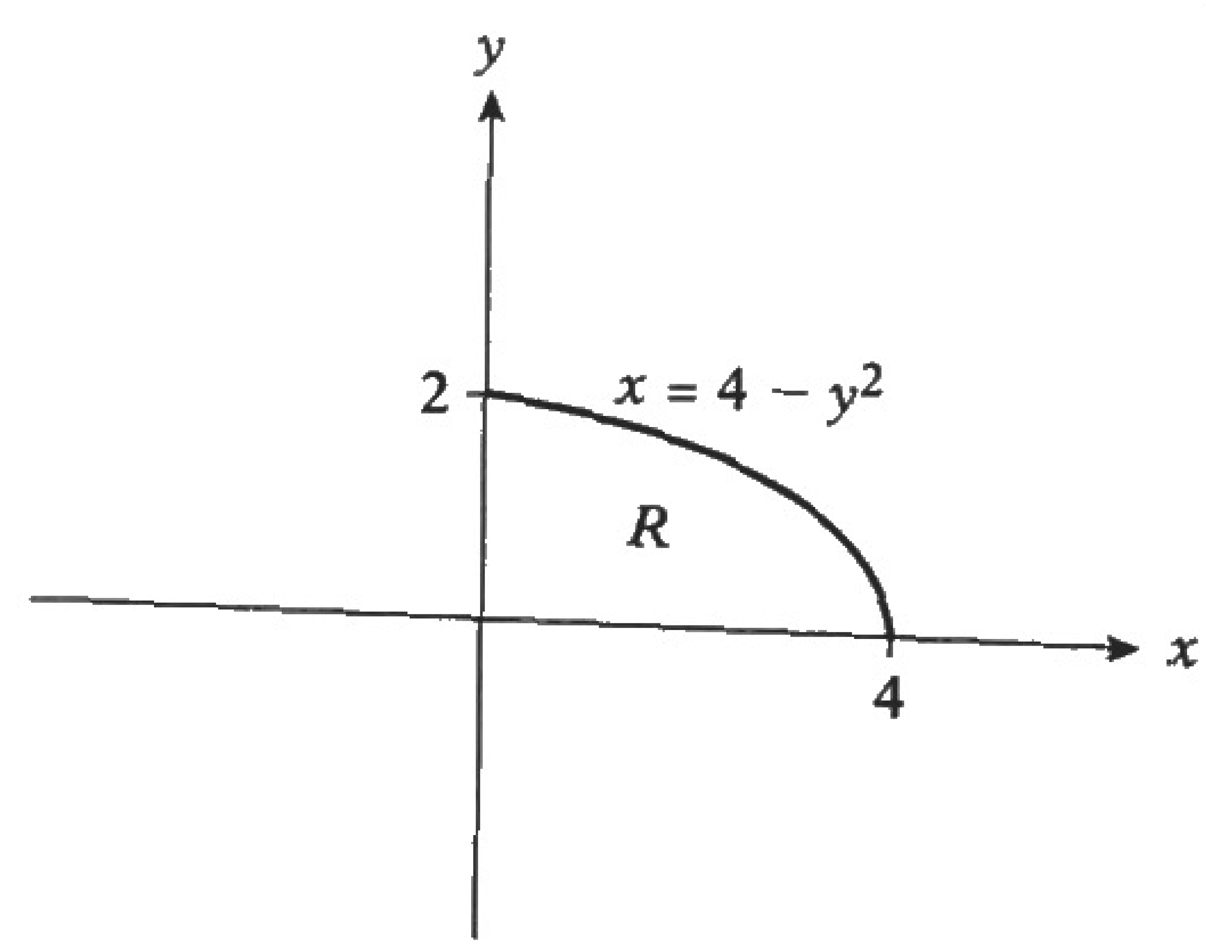
I a relative minimum at $x = \frac{1}{2}$
II a horizontal asymptote $y = 0$
III a vertical asymptote $x = 0$

(A) I only
(B) I and II
(C) I and III
(D) II and III
(E) I, II, and III

5. Find the shortest distance from (3, 0) to a point on the curve $y = x^2 - 2x$.

(A) 0.908
(B) 1.0
(C) 2.165
(D) 2.20
(E) 3.0

6. A solid is formed that has the region R as its base and cross sections perpendicular to the x-axis that are squares. Find the value of k so that the volume of the solid on the interval $[0, k]$ is half the total volume of the solid.

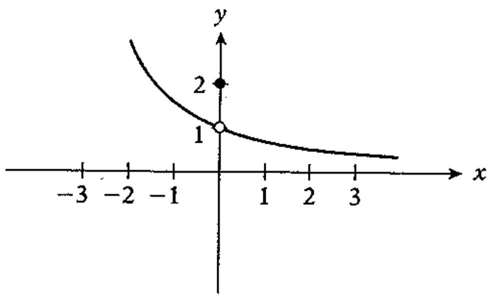


(A) 0.568
(B) 1.172
(C) 2.201
(D) 3.2
(E) 3.567

7. $\frac{d}{dx}\left(\lim_{h \to 0} \frac{\ln(x+h) - \ln x}{h}\right) =$

(A) $-\frac{1}{x^2}$
(B) $\frac{1}{x}$
(C) -1
(D) 0
(E) undefined

8. Assuming that the function graphed here behaves like the function e^{-x} outside of the domain shown, which of the following is false?



(A) $\lim_{x \to 0} g(x) = 1$
(B) $\lim_{x \to \infty} g(x) = 0$
(C) $g'(x) < 0$ for $x \neq 0$
(D) $g'(0) = 2$
(E) $g''(x) > 0$ for $x \neq 0$

9. For what value(s) of x are the lines tangent to $f(x) = \frac{1}{3}x^3 + 5$ and $g(x) = 4 + 2x - \frac{x^2}{2}$ parallel?

(A) $x = -2$ and $x = 1$
(B) $x = -2$ only
(C) $x = 1$ only
(D) $x = -3.475$ only
(E) $x = 4$ only

10. The equation of the line normal to $y = \frac{x^2}{x^2 + 1}$ at $x = 1$ is

(A) $y = -2x$
(B) $y = -2x - 1$
(C) $y = -2x + 2.5$
(D) $y = \frac{1}{2}x$
(E) $y = \frac{1}{2}x + 1$

11. A contractor is building a rectangular house that is 3,000 ft^2. What dimensions will require the least amount of building materials?

(A) $w = 31.65228$ ft, $l = 94.8683$ ft
(B) $w = 109.5445$ ft, $l = 27.3861$ ft
(C) $w = 54.7723$ ft, $l = 54.7723$ ft
(D) $w = 38.7298$ ft, $l = 77.4597$ ft
(E) $w = 98.9867$ ft, $l = 30.3071$ ft

12. $f(x) = \sin x$. Which of the following are true?

I $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

II $\int_{-a}^{a} f(x)\,dx = 0$

III $\int_{a}^{c} f(x)\,dx + \int_{c}^{b} f(x)\,dx = \int_{a}^{b} f(x)\,dx$, for $a < c < b$.

(A) I only
(B) II only
(C) III only
(D) I and III
(E) II and III

13. The derivative of $y(x) = \arcsin \frac{x}{2}$ on $-1 < x < 1$ is

(A) $y = \dfrac{1}{2\sqrt{1 - \frac{x^2}{4}}}$

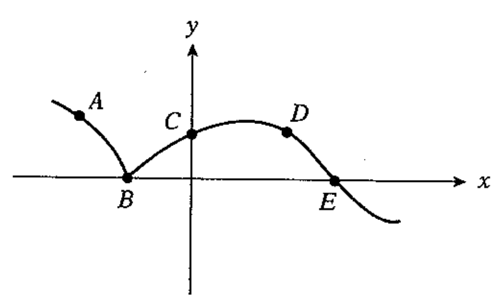
(B) $y = \dfrac{1}{2\sqrt{1 - \sin(x)}}$

(C) $y = \dfrac{1}{2\cos\left(\arcsin \frac{x}{2}\right)}$

(D) $y = \dfrac{\arccos \frac{x}{2}}{2}$

(E) $y = -\dfrac{\arccos \frac{x}{2}}{2}$

14. In the graph shown, at which point is $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?



(A) A
(B) B
(C) C
(D) D
(E) E

15. If $f(x) = \begin{cases} x^2 + 2, & x < 1 \\ 2x + 1, & x \geq 1 \end{cases}$, which of the following is true about $f(x)$?

(A) The function is not continuous and not differentiable at $x = 1$.
(B) The function is continuous, but not differentiable at $x = 1$.
(C) The function is differentiable, but not continuous at $x = 1$.
(D) The function is both continuous and differentiable at $x = 1$.
(E) The function is not integratable.

16. If $f(x) = 3x^2 - 4$, then $\lim_{x \to a} \frac{f'(x) - f'(a)}{x - a} =$

(A) $3a^2 - 4$
(B) $6a$
(C) 6
(D) 0
(E) does not exist

17. Find the area of the region enclosed by the semicircle $y = \sqrt{16 - x^2}$ and the line $y = 2$.

(A) 0.913
(B) 3.653
(C) 4.913
(D) 7.306
(E) 9.827