AB Model Examination 2

Section I
Part A (pages 293–296)

1. (B) (1, 1)

$f'(x) = 6x^2 - 12x + 6$

$f''(x) = 12(x - 1)$

$x = 1$

2. (B) $\frac{7}{10}$

$\frac{1}{b-a}\int_a^b f(x)dx$

$\frac{1}{4-(-1)}\left(\left(\int_{-1}^{1} x + 1\right) + \left(\int_1^4 -x + 3\right)\right)$

3. (D) -1

$$\int_{\frac{\pi}{2}}^{\pi} 2\sin(x)\cos(x)dx = -\frac{\sin^2(x)}{2}\Big|_{\frac{\pi}{2}}^{\pi} = -1$$

4. (A)

5. (B) $-\frac{1}{12}$

$\frac{dy}{dx} = \frac{-3y + 2}{3x + 6y}$

$\frac{-3(1) + 2}{3(2) + 6(1)} = -\frac{1}{12}$

6. (D) 1

7. (D) $\frac{1}{(x+2)^2}$

8. (D) 4

9. (E) does not exist

10. (C) $2\sin x \cos x e^{\sin^2 x}$

11. (B) 1

12. (B) 1

$x'(t) = 3t^2 - 2t - 1$

$3t^2 - 2t - 1 = 0$

$(t - 1)(3t + 1) = 0$

$t = 1$

13. (D) $x = 1$ and $x = 2$

14. (B) $1 - \frac{\pi}{2}$

$$\int_0^1 \frac{x^2 - 1}{x^2 + 1} = \int_0^1 \frac{x^2}{x^2 + 1} - \int_0^1 \frac{1}{x^2 + 1}$$

$$x - \arctan(x) - \arctan(x)\Big|_0^1$$

$$= x - 2\arctan(x)\Big|_0^1 = 1 - \frac{\pi}{2}$$

15. (C) $2\sqrt[3]{1 + 2x}$

$$\int_2^{2x} (1 + t)^{1/3} = \frac{3(1 + t)^{4/3}}{4}\Big|_2^{2x}$$

$$= \frac{3(1 + 2x)^{4/3}}{4} - \frac{3(1 + 2)^{4/3}}{4}$$

$f'(x) = 2\sqrt[3]{1 + 2x}$

16. (D)

17. (A) $y - 3 = 8\sqrt{3}\left(x - \frac{\pi}{3}\right)$

$f'(x) = \frac{4\sin 2x}{(\cos 2x + 1)^2}$

slope $= 8\sqrt{3}$

$y - 3 = 8\sqrt{3}\left(x - \frac{\pi}{3}\right)$

18. (A) A relative maximum at -1 and a relative minimum at 0. Use the First Derivative Test.

19. (A) $p = \frac{1}{a}$

$$\int_1^a \frac{1}{x}dx = \int_p^1 \frac{1}{x}$$

$\ln(a) = -\ln(p)$

$-\ln(a) = \ln(p)$

$\ln\left(\frac{1}{a}\right) = \ln(p)$

20. (D) $y = e^{k(t-1)}$

$$\frac{dt}{dy} = \frac{1}{ky}$$

$$t = \frac{1}{k}\int \frac{dy}{y}$$

$$t = \frac{1}{k}\ln(y) + C$$

$$kt - C = \ln(y)$$

$$e^{kt-C} = y(t)$$

$e^{k-C} = 1$; therefore, $C = k$.

21. (C) $(0, \infty)$

22. (E) 18

$v(t) = 3t^2 - 12t + 18$

$v(t) = 18$ at $t = 0$ and $t = 4$. (It reaches its minimum velocity at $t = 2$.)

23. (B) $3(2x + 5)^2$

24. (B) $\int_0^2 (2x - x^2)\,dx$

$x^2 - 4 = 2x - 4$

$x = 0$ or $x = 2$

25. (D) 2

$$\lim_{x\to\infty} \frac{2x}{\sqrt{x^2+1}} = \frac{\frac{2x}{2}}{\frac{\sqrt{x^2+1}}{\sqrt{x^2}}} = \frac{2}{1+\frac{1}{x^2}}$$

26. (B) For every k between $g(a)$ and $g(b)$, there is a value c in $[a, b]$ such that $g(c) = k$.

27. (B) $\pi\int_0^8 (4 - x^{2/3})^2 dx$

$V = \pi r^2 \Delta x$ (disc method)

$r = 4 - x^{2/3}$

$x^{2/3} = 4$ when $x = 8$

28. (D)

Section I
Part B (pages 297–299)

1. (E) undefined

2. (C) 1

$$\frac{dy}{dx} = \frac{2t+1}{t^2+t+1}$$

3. (A) 0

$$y' = \frac{8\sin(4x)}{\cos^2(4x)+1}$$

$$\frac{8\sin\left(4\times\left(\frac{\pi}{2}\right)\right)}{\cos^2\left(4\times\left(\frac{\pi}{2}\right)\right)+1} = \frac{0}{4} = 0$$

4. (E)

5. (A) 0.908

$D = \sqrt{(x-3)^2 + (x^2 - 2x - 0)^2}$

$f(x) = x^4 - 4x^3 + 5x^2 - 6x + 9$

$f'(x) = 4x^3 - 12x^2 + 10x - 6$

$x = 2.165, y = 0.358$

$D = \sqrt{(2.165-3)^2 + (0.358-0)^2} = 0.908$

6. (B) 1.172

$y = \sqrt{4-x}$

$\int_0^4 \left(\sqrt{4-x}\right)^2 dx$

$= 4x - \frac{x^2}{2}\Big|_0^4 = 8$

$4x - \frac{x^2}{2} = 4$

$x = 1.172$

7. (A) $-\frac{1}{x^2}$

8. (D)

9. (A) $x = -2$ and $x = 1$

$f'(x) = x^2$ and $g'(x) = 2 - x$

Parallel lines have equal slopes.

$x^2 = 2 - x$

$x = 1$ and $x = -2$

10. (C) $y = -2x + 2.5$

11. (C) $w = 54.7723$ ft, $l = 54.7723$ ft

$A = lw$

$l = \frac{3{,}000}{w}$

$P = 2w + 2\left(\frac{3{,}000}{w}\right)$

$P' = 2 - \left(\frac{6{,}000}{w^2}\right) = 0$

$w = 10\sqrt{30} = 54.7723$

12. (E) II and III

13. (A) $y = \dfrac{1}{2\sqrt{1 - \frac{x^2}{4}}}$

14. (C) C

15. (D)

16. (C)

17. (E)