KEY

Name ____________________

Period ____________________

Calculus BC – Semester 1 Final Review

1. The position of a car is given by the values in the table:

t (seconds)	0	1	2	3	4	5
s (feet)	0	10	32	70	119	178

3: 48.667 2: 43.5 1: 38

(A) Find the average velocity for the time period beginning when $t = 2$ and lasting: 3 sec., 2 sec., 1 sec.
(B) Use the information from part (A) and other calculations to approximate the instantaneous velocity when $t = 2$. 30

2. Calculate derivative:
(A) $y = (x+2)^8 (x+3)^6$ $y' = 8(x+2)^7(x+3)^6 + 6(x+3)^5(x+2)^8$
(B) $y = \left(x + \frac{1}{x^2}\right)^5$ $y' = 5\left(x + \frac{1}{x^2}\right)^4\left(1 - \frac{2}{x^3}\right)$
(C) If $f(x) = \dfrac{1}{(2x-1)^{\sqrt{7}}}$, find $f''(1)$. $28 + 4\sqrt{7}$

3. Calculate derivative:
(A) $y = \cos(3x^2 + 5)$ $y' = -6x \sin(3x^2+5)$
(B) $y = \sin^{-1}(e^x)$ $y' = \dfrac{e^x}{\sqrt{1-e^{2x}}}$
(C) $y = (x + \sin x)^\pi$ $y' = \pi(x + \sin x)^{\pi - 1}(1 + \cos x)$

4. Calculate derivative:
(A) $y = \csc(x - \sin x)$ $y' = -\csc(x - \sin x)\cot(x - \sin x)(1 - \cos x)$
(B) $y = \ln(\sec^2 x)$ $y' = 2\tan x$
(C) If $f(x) = 2^x$, find $f'''(x)$. $f'''(x) = (\ln 2)^3 2^x$

5. The table below gives values for the functions f and g, as well as their derivatives.

x	-1	0	1	2	3
f	3	3	1	0	1
g	1	2	2.5	3	4
f'	-3	-2	-1.5	-1	1
g'	2	3	2	2.5	3

(A) Find $\frac{d}{dx} f(x)g(x)$ and $\frac{d}{dx}\frac{f(x)}{g(x)}$ at $x = -1$. $\frac{d}{dx}(f \cdot g) = 3$ $\frac{d}{dx}\left(\frac{f}{g}\right) = -9$

(B) Find $\frac{d}{dx} f(g(x))$ and $\frac{d}{dx} g(f(x))$ at $x = 0$. $\frac{d}{dx} f(g) = -3$ $\frac{d}{dx} g(f) = -6$

6. Find $\frac{dy}{dx}$:
(A) $x^2 y + xy^2 = 3x$ $\dfrac{dy}{dx} = \dfrac{3 - y^2 - 2xy}{x^2 + 2xy}$
(B) $\sin(ax) + \cos(by) = xy$ (a and b are constants) $\dfrac{dy}{dx} = \dfrac{a\cos(ax) - y}{b\sin(by) + x}$
(C) Find the slope of the curve $x^2 + 3y^2 = 7$ at the point (-2, 1). $m = 2/3$

7. Find all local max./min. points and inflection points for $y = x - 2\ln x$, $x > 0$. $x = 2$ min., no inf.

8. Find all local max./min. points and inflection points for $y = x^2 e^{5x}$. $x = 0$ min, $x = -2/5$ max; $x = -.683$, $x = -.117$ inf.

9. Find the absolute max./min. values of $y = \frac{x}{x^2+1}$ on the interval [0, 2]. max $= \frac{1}{2}$ min $= 0$

10. The volume of a cube is increasing at a rate of 10 cm^3/min. How fast is the surface area increasing when the length of an edge is 30 cm.? $\frac{4}{3}$ cm^2/min

11. A balloon is rising at a constant rate of 5 ft./sec. A boy is cycling along a straight road at a speed of 15 ft./sec. When he passes under the balloon it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 sec. later? 13 ft/sec.

12. The angle of elevation of the sun is decreasing at a rate of 0.25 rad./hr. How fast is the shadow cast by a 400-ft-tall building increasing when the angle of elevation is $\frac{\pi}{6}$? 400 ft/hr.

13. A 10 ft. ladder is leaning up against the side of a building, but slipping so that the top of the ladder is descending at a rate of 3 ft./sec. What is the rate at which the base of the ladder moves away from the building when the top is 6 ft. from the ground? $\frac{9}{4}$ ft/sec.

14. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible. 500, 125

15. The top and bottom margins of a poster are each 6 cm. and the side margins are each 4 cm. If the area of printed material on the poster is 384 cm^2, find the dimensions of the entire poster with the smallest area. 36 cm. x 24 cm.

16. Limits

(A) $\lim\limits_{x\to 0}\frac{\sin(a+2x)-2\sin(a+x)+\sin a}{x^2}$ $= -\sin a$

(B) $\lim\limits_{x\to\infty}\frac{x}{\ln(1+2e^x)}$ $= 1$

(C) $\lim\limits_{x\to\pi}\frac{(x-\pi)}{\tan x}$ $= 1$

17. Riemann Sums

(A) The value of $\int_0^2 (x^2-x)\,dx$ is estimated using 4 subintervals. Find the estimates using left- and right-hand sums.

LHS $= \frac{1}{4}$
RHS $= \frac{5}{4}$

(B) Consider the function with values given in the table below. Approximate $\int_0^6 f(x)\,dx$ using left- and right-hand Riemann sums with 6 subintervals.

x	0	1	2	3	4	5	6
$f(x)$	1	2.5	3	1	-2	-3.5	-4

LHS $= 2$
RHS $= -3$

18. Area under a curve

(A) Find the area under one arch of the curve $y = \sin x$. 2

(B) Assume $f(x)$ is a positive function on the interval [2,5]. If $\int_2^5 (2f(x)+3)\,dx = 17$, find the area under the curve $f(x)$ on the interval [2,5]. 4

(C) Find the area bounded by $y = x^2 - 9$ and the x-axis. 36

(D) Without a calculator, compute $\int_{-1}^{1}\sqrt{1-x^2}\,dx$. $\pi/2$

19. Average value of a function:

76.817 mil

(A) Assume the population, P, of Mexico (in millions), is given by $P = 67.38(1.026)^t$, where t is the number of years since 1980. What was the average population of Mexico between 1980 and 1990?

(B) A bar of metal is heated, and then allowed to cool. The temperature, H, of the bar t minutes after it starts cooling is given, in °C, by $H = 20 + 980e^{-0.1t}$. Find the average value of the temperature over the first hour. 182.928 °C

20. Fundamental Theorem of Calculus

(A) The rate at which the world's oil is being consumed is continuously increasing. Suppose the rate (in billions of barrels per year) is given by the function $r = 32e^{0.05t}$, where t is measured in years since 1990. Find the total quantity of oil used between 1990 and 1995. 181.776 bil.

(B) Assume that $r(t)$ represents the rate at which a country's debt is growing, where t is years since 1990. In terms of debt, explain the meaning of $\int_0^{10} r(t)\,dt$. increase in debt from 1990 to 2000

21. Find the total area bounded by the curve $f(x) = x(x-4)(x-1)$ and the x-axis. $71/6$

22. Find an antiderivative of each of the following:

(A) $\pi + x^2 + \dfrac{1}{\pi x^2}$ $\quad \pi x + \frac{1}{3}x^3 - \frac{1}{\pi x}$

(B) $\sqrt{x} - \dfrac{1}{x\sqrt{x}}$ $\quad \frac{2}{3}x^{3/2} + \frac{2}{\sqrt{x}}$

(C) $\cos(2\theta)$ $\quad \frac{1}{2}\sin(2\theta)$

(D) $e^t + e^3$ $\quad e^t + e^3 t$

(E) $\dfrac{2}{x} + \dfrac{x}{2}$ $\quad 2\ln|x| + \frac{x^2}{4}$

23. On Planet Janet, the gravitational constant g is -15 feet per second per second. A ball is launched upward from the ground at time $t = 0$ at 60 feet per second.

(A) Find the equations for the ball's acceleration a, velocity v, and position s. $a = -15$, $v = -15t + 60$, $s = -\frac{15}{2}t^2 + 60t$

(B) Find the peak height of the ball. 120 ft

(C) On Planet Nanette, g is -5. Find the peak height of the ball. 360 ft.

24. Evaluate the following:

(A) $\int (x^2 + e^{3x})(x^3 + e^{3x})^{4/5}\,dx$ $\quad \frac{5}{27}(x^3 + e^{3x})^{9/5} + C$

(B) $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx$ $\quad \sin(\ln x) + C$

(C) $\displaystyle\int \frac{8e^{=2w}}{6 - 5e^{-2w}}\,dw$ $\quad \frac{4}{5}\ln|6 - 5e^{-2w}| + C$

(D) $\displaystyle\frac{d}{dx}\int_e^x \log_5(t^{21})\sin(\sqrt{t})\,dt$ $\quad \log_5(x^{21})\sin(\sqrt{x})$

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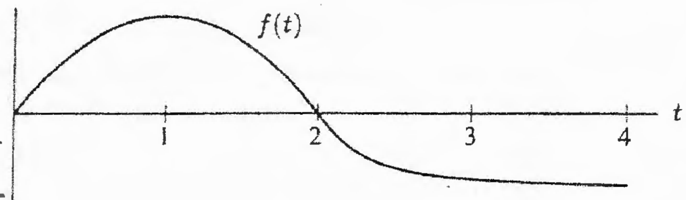
25. The function $f(t)$ is graphed at right and we define $F(x)=\int_0^x f(t)\,dt$. Are the following statements true or false? Justify your answer.



(A) $F(x)$ is positive for all x between 2 and 3. T

(B) $F(x)$ is decreasing for all x between 1 and 3. F

(C) $F(x)$ is concave down at $x=\frac{1}{2}$. F

26. Evaluate the following:

(A) $\int 4x^5\,dx$ $\quad \frac{2}{3}x^6 + C$

(B) $\int \frac{x^2-x+1}{x}\,dx$ $\quad \frac{1}{2}x^2 - x + \ln|x| + C$

(C) $\int e^x \cos(e^x)\,dx$ $\quad \sin(e^x) + C$

27. Find an antiderivative for each of the following:

(A) $x^2 - \frac{3}{x} + \frac{2}{x^3}$ $\quad \frac{1}{3}x^3 - 3\ln|x| - \frac{1}{x^2}$

(B) $\left(x+\sqrt{\sin(2x+3)}\right)\left(x-\sqrt{\sin(2x+3)}\right)$ $\quad \frac{1}{3}x^3 + \frac{1}{2}\cos(2x+3)$

28. Find $\int x^2 \sin 5x\,dx$ $\quad -\frac{1}{5}x^2\cos 5x + \frac{2}{25}x\sin 5x + \frac{2}{125}\cos 5x + C$

29. Evaluate the following:

(A) $\int \sec^2\theta\,d\theta$ $\quad \tan\theta + C$

(B) $\int ze^{3z^2+1}\,dz$ $\quad \frac{1}{6}e^{3z^2+1} + C$

30. Evaluate the following:

(A) $\int y\sec^2 y\,dy$ $\quad y\tan y - \ln|\sec y| + C$

(B) $\int \frac{dx}{(b+ax)^2}$ $\quad a, b$ constants $\quad -\frac{1}{a}\cdot\frac{1}{b+ax} + C$

31. Evaluate the following:

(A) $\int \frac{x^3+1}{x^2}\,dx$ $\quad \frac{1}{2}x^2 - \frac{1}{x} + C$

(B) $\int \frac{x^2}{x^3+1}\,dx$ $\quad \frac{1}{3}\ln|x^3+1| + C$

(C) $\int \frac{\sqrt{\ln x}}{x}\,dx$ $\quad \frac{2}{3}(\ln x)^{3/2} + C$

(D) $\int \sin(3x)e^{\cos(3x)}\,dx$ $\quad -\frac{1}{3}e^{\cos(3x)} + C$

32. The following are some of the values for a function $G(x)$.

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$G(x)$	0	0.1	0.199	0.296	0.390	0.480	0.567	0.649	0.726	0.798	.0866

Use these values to approximate the value of $\int_0^1 G(x)\,dx$ using the Midpoint Rule with 5 subintervals. .4646

33. Consider the differential equation $\frac{dy}{dx} = x^2 + y$.

(A) Use Euler's method with two steps to approximate the value of y when $x = 2$ on the solution curve that passes though the point $(1,3)$. 8.625

(B) Is your approximate value of y an under- or over-estimate of the exact value? Justify your answer. under, $y'' > 0$

34. Solve the initial value problem:

(A) $\frac{dy}{dx} = \frac{x}{y}$, $y(1) = 3$ $\quad y = \sqrt{x^2 + 8}$

(B) $\frac{dy}{dx} = y^2 + 1$, $y(0) = 1$ $\quad y = \tan\left(x + \frac{\pi}{4}\right)$

35. Solve the initial value problem $\frac{dy}{dx} = \frac{5}{1+y}$, $y(0) = 2$ $\quad y + \frac{1}{2}y^2 = 5x + 4$

36. Solve the initial value problem:

(A) $\frac{dy}{dx} = xy$, $y(0) = 5$ $\quad y = 5e^{\frac{1}{2}x^2}$

(B) $\frac{dy}{dx} = x\sec y$, $y(1) = \frac{\pi}{6}$ $\quad y = \sin^{-1}\left(\frac{1}{2}x^2\right)$

37. Solve the initial value problem $\frac{dy}{dx} = \frac{\cos^2 y}{x}$, $y(1) = \frac{\pi}{4}$ $\quad y = \tan^{-1}\left[\ln|x| + 1\right]$

38. Consider the differential equation $\frac{dP}{dt} = 0.0005P(1000 - P)$ with initial condition $P(0) = 1$.

(A) Find the size of the population P when the rate of chance of the population is a maximum. $P = 500$

~~(B) Find the size of the population P when the rate of chance of the population is a minimum.~~

(C) Sketch a graph of the population as a function of time.

(D) Find $\lim_{t\to\infty} P(t)$. 1000

39. Consider the region bounded by $y = e^x$, the x-axis, and the lines $x = 0$ and $x = 1$. Find the volume of the solid whose base is the given region and whose cross sections perpendicular to the x-axis are isosceles right triangles with hypotenuse lying in the region. .799

40. What is the arc length of $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 4$? 6.787