BC Model Examination 2

Section I

Part A

No calculator is allowed for these questions.

2. Write the equation of the line tangent to the graph of $y = \frac{x+3}{x}$ at $x = 1$.
   (A) $3x + y = 7$
   (B) $-3x + y = 1$
   (C) $3x + y = 13$
   (D) $x - 3y + 11 = 0$
   (E) $3y = 13 - x$

3. For what value of c does $y = cx + \frac{3}{x^2}$ have a relative minimum at $x = 2$?
   (A) $-\frac{3}{4}$
   (B) $-\frac{3}{8}$
   (C) 0
   (D) $\frac{3}{4}$
   (E) 6

4. $f(x) = \frac{5x}{x-5}$. Find the average rate of change of $f'(x)$ on the closed interval $[0, 4]$.
   (A) -12.6
   (B) $-\frac{13}{2}$
   (C) -6
   (D) -5
   (E) 6

5. $\int_0^{\infty} \frac{\ln(x)}{x}\,dx =$
   (A) -1
   (B) 0
   (C) $\frac{1}{2}$
   (D) 1
   (E) diverges

6. For what values of a and c is the function $f(x) = \begin{cases} ax^2, & x \le 2 \\ x + c, & x > 2 \end{cases}$ differentiable for all real values of x?
   (A) $a = \frac{1}{2}, c = 0$
   (B) $a = \frac{1}{4}, c = -1$
   (C) $a = 1, c = 6$
   (D) $a = 0, c = -2$
   (E) for no values of a and c

7. $\lim_{x \to \infty} \frac{\ln(\ln(x))}{\ln(x)} =$
   (A) diverges
   (B) 0
   (C) $\frac{1}{2}$
   (D) 1
   (E) e

8. $\lim_{x \to \infty} x^5 e^{-x/5} =$

(A) $-\infty$

(B) -1

(C) 0

(D) 1

(E) ∞

9. $\frac{dx}{dt} = 5x^2$ and $x = 5$ when $t = 0$. Find the value of x when $t = 1$.

(A) $-\frac{5}{24}$

(B) $\frac{5}{24}$

(C) $\frac{1}{5}e$

(D) $\frac{5}{4}$

(E) $5e$

10. A particle travels on a number line with velocity $v(t) = t \cos t$. Find the distance traveled from $t = 0$ to $t = \frac{\pi}{2}$.

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{2} - 1$

(C) $\frac{\pi}{2} + 1$

(D) 1

(E) $1 - \frac{2}{\pi}$

11. Given $\frac{dy}{dx} = x^2y + x^2 + y + 1$, and $y(0) = 0$, then $y =$

(A) $e^{\frac{x^2}{2}+1} - 1$

(B) $\frac{x^3}{3} + x$

(C) $e^{\frac{x^3}{3}+x} - 1$

(D) $e^{\frac{x^3}{3}+x-1}$

(E) $\ln\left(\frac{x^3}{3} + x\right) - 1$

12. $f(x) = \ln|x^2 - 2x|$. $f'(x) =$

(A) $\frac{2x-2}{x^2-2x}$

(B) $\left|\frac{2x-2}{x^2-2x}\right|$

(C) $\frac{|2x-2|}{x^2-2x}$

(D) $\frac{2x-2}{|x^2-2x|}$

(E) does not exist

13. Find the interval of convergence of the series $\sum_{n=1}^{\infty}\left(\frac{x}{n}\right)^n$, where x is a real number.

(A) $[0, \infty)$

(B) $[-1, 0]$

(C) $[-1, 1]$

(D) $[0, 1]$

(E) $(-\infty, \infty)$

14. What function is approximated by the following series?

$$(x-2) - \frac{(x-2)^2}{2} + \frac{(x-2)^3}{3} - \ldots$$

(A) e^{x-2}

(B) $\ln(x-2)$

(C) $\ln(x-1)$

(D) $\sin(x-2)$

(E) $\cos(x-2)$

15. $\frac{dy}{dx} = 5e^{-y}$ and $y(0) = 0$. Find the value of $\frac{d^2y}{dx^2}$ at $x = 0$.

(A) -25

(B) -5

(C) 0

(D) 5

(E) 25

16. A company has 1,000 widgets and will be able to sell them all if the price is a dollar. The company will sell one less widget for each 10-cent increase in the price it charges. What price will maximize revenues, where revenue is the selling price times the quantity sold?

(A) \$2.50

(B) \$14.50

(C) \$22.30

(D) \$25.30

(E) \$50.50

17. $\int x^2\sqrt{x+1}\,dx =$

(A) $\frac{2}{7}(x+1)^{7/2} - \frac{4}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C$

(B) $\frac{2}{7}(x+1)^{7/2} - \frac{2}{5}(x+1)^{5/2} + \frac{4}{3}(x+1)^{3/2} + C$

(C) $\frac{10}{7}(x+1)^{7/2} - 2x^{1/2} + C$

(D) $(x+1)^{3/2}\left(\frac{4}{5}x^2 - x + 1\right) + C$

(E) $(x+1)^{3/2}\left(\frac{2}{7}x^2 - x + 1\right) + C$

20. Which of the following is the series representation for $\frac{1}{1+5x}$?

(A) $\sum_{n=0}^{\infty}(-1)^n(5x)^n$

(B) $\sum_{n=0}^{\infty}(-1)^{n+1}(5x)^n$

(C) $\sum_{n=0}^{\infty}(-1)^n\frac{(5x)^n}{n!}$

(D) $\sum_{n=0}^{\infty}\frac{(5x)^n}{n}$

(E) $\sum_{n=1}^{\infty}(-1)^n(5x)^n$

21. If the motion of a particle on a number line is described by $a(t) = 2t + 2$ and $v(1) = 4$ for $t \geq 0$, find the distance traveled by the particle in the first three seconds.

(A) 8

(B) 15

(C) 21

(D) 24

(E) cannot be determined

22. Find the equation of the line tangent to $y = (\arctan x)^2$ at $x = 1$.

(A) $y = \frac{\pi}{4x} + \frac{\pi^2}{16}$

(B) $y = \frac{\pi}{4x} - \frac{\pi^2}{16}$

(C) $y = \frac{\pi}{4x} + \frac{\pi(\pi-4)}{16}$

(D) $y = \frac{\pi}{4}$

(E) does not exist

23. Find the area of the inner loop of the polar curve $r = 1 + 2\cos\theta$.

(A) 2π

(B) $2\pi - 3\sqrt{3}$

(C) 3π

(D) $\pi + \frac{3\sqrt{3}}{2}$

(E) $\pi - \frac{3\sqrt{3}}{2}$

24. $g(x) = \int_e^x \ln(1+t)\,dt$. Find the derivative of $(g'(x))^2$ at $x = e - 1$.

(A) 0

(B) $\frac{2}{e}$

(C) 1

(D) 2

(E) does not exist

26. The voltage in a particular electrical circuit is $v(t) = 2\sin(3\pi t)$. The current through the circuit is $i(t) = \frac{dv}{dt}$. If the power consumed by the circuit is $p(t) = i(t)v(t)$, find the average power consumed for $0 < t < \pi$.

(A) 0

(B) $\frac{2}{\pi}(\sin^2(3\pi^2))$

(C) $-\frac{2}{\pi}(\sin^2(3\pi^2))$

(D) $\frac{4}{\pi}(\sin^2(3\pi^2))$

(E) $\frac{2}{3\pi}(\sin^2(3\pi^2))$

27. F is the antiderivative of $f(x) = -2x^2 + 4x$. If $F(0) = 5$, then $F(1) =$

(A) $\frac{4}{3}$

(B) $\frac{7}{3}$

(C) $\frac{11}{3}$

(D) $\frac{19}{3}$

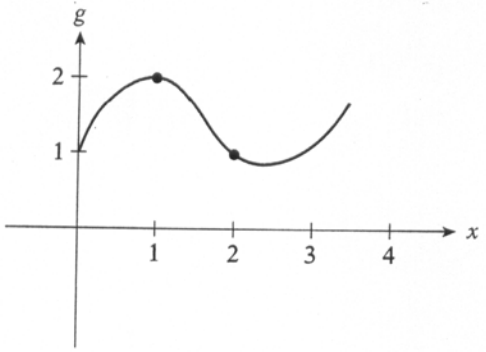
(E) $\frac{23}{3}$

Section I

Part B

A graphing calculator is required for some questions.

2. In the graph of g, $g = f'$ and $f(0) = 2$.



Which of the following are true?

I $g(1) > g(2)$
II $f(1) < f(2)$
III $f''(3) > 0$

(A) I only
(B) II only
(C) I and II
(D) I and III
(E) I, II, and III

3. Find the area in the first quadrant bounded by the graphs of $y = \frac{1}{x} - 1$, $y = 2x$, and the x-axis.

(A) 0.443
(B) 0.5
(C) 0.886
(D) 1
(E) does not exist

4. Find the length of the curve described by the parametric equations $x(t) = 2\sin(t)$ and $y(t) = 2\ln(\sin t)$ on $0.1 \le t \le 1.0$.

(A) 2.042
(B) 3.243
(C) 4.491
(D) 4.587
(E) undefined

5. If f is differentiable on (a, b), continuous on $[a, b]$, and $f(a) = f(b)$, which of the following statements could be false?

(A) $\lim_{x \to x_0} f(x)$ exists for all $a < x_0 < b$.
(B) f has a point of inflection in $[a, b]$.
(C) f has a maximum.
(D) There exists $c > a$ and $c < b$ such that $f'(c) = 0$.
(E) There exists $c > a$ and $c < b$ such that $f(b) - f(a) = f'(c)(b - a)$.

7. $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx =$

(A) $-\frac{\pi}{2}$

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{\pi}{2}$

(E) not defined

8. The area of $r = 2 + \cos(\theta)$ on $0 < \theta < \pi$ is

(A) 2π

(B) $2\pi + \frac{\pi}{3}$

(C) π

(D) $\frac{5\pi}{4}$

(E) $\frac{9\pi}{4}$

9. $f(x) = 4x - x^3$ on the interval $[0, 2]$. Find the point of intersection of the lines tangent to the graph of f at the endpoints of the interval.

(A) (1.333, 5.333)

(B) (1, 3)

(C) (1, 4)

(D) (0.75, 3.578)

(E) (1.155, 3.079)

10. Let $f(x) = xe^{x^2}$. If g is the inverse of f, then $g'(e) =$

(A) 0

(B) $\frac{1}{4e}$

(C) $\frac{1}{3e}$

(D) $\frac{1}{e}$

(E) e

11. Use the Trapezoidal Rule with five equal subintervals to approximate $\int_0^1 \ln(x^2 + 1)\,dx$.

(A) 0.264

(B) 0.267

(C) 0.337

(D) 0.385

(E) 0.534

12. $y' = xy + 2y$ and $y(0) = 1$. What is the result when you use Euler's method with $\Delta x = 0.1$ to approximate $y(0.3)$?

(A) 1.452

(B) 1.771

(C) 1.906

(D) 2.338

(E) 2.411

13. If $h(x) = \sqrt{\sin(f(x))}$, $f(0) = \frac{\pi}{3}$, and $f'(0) = e$, approximate $h'(0)$.

(A) −1.6864

(B) −0.84321

(C) −0.3102

(D) 0.6409

(E) 0.73024

14. Water leaks from a storage tank at a rate $R(t) = 2te^{-0.5t}$, in thousands of gallons per day, $t \geq 0$. How many gallons have leaked at the end of the first week, to the nearest gallon?

(A) 3,456 gallons

(B) 6,191 gallons

(C) 6,407 gallons

(D) 6,913 gallons

(E) 13,826 gallons

15. If $y = 2x(x - 2)^2 - p$, for how many integer values of p does y have three distinct zeros?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

16. $g(x) = \int_0^x (\ln(1 + t))^2\,dt$. Find $g'(e - 1)$.

(A) 1

(B) $\frac{2}{e}$

(C) $\frac{2\ln(1+e)}{e}$

(D) $\frac{2\ln(1+e)}{e+1}$

(E) $\frac{2}{e+1}$

17. Which of the following series are conditionally convergent?

I $\sum_{0}^{\infty} \frac{(-1)^n}{n!}$

II $\sum_{1}^{\infty} \frac{(-1)^{n+1} n^2}{n^2 + n}$

III $\sum_{0}^{\infty} \frac{(-1)^{n+1} 3^n}{5^n}$

(A) none
(B) I only
(C) I and II
(D) II and III
(E) I, II, and III

MC-1 BC

1.
2. A
3. D
4. C
5. E
6. B
7. B
8. C
9. A
10. B
11. C
12. A
13. E
14. C
15. A
16. E
17. A
18.
19.
20. A
21. C
22. C
23. E
24. B
25.
26. B
27. D
28.

1.
2. E
3. A
4. D
5. B
6.
7. D
8. E
9. A
10. C
11. B
12. B
13. E
14. D
15. C
16. A
17. A