Name _____

Period _____

Calculus BC – Chapter I Sample Test (calculators allowed)

Show all work for free-response questions.

1. Let f be a differentiable function such that $\int f(x)\sin x dx = -f(x)\cos x + \int 4x^3 \cos x dx$.

   Which of the following could be $f(x)$?

   (A) $\cos x$ (B) $\sin x$ (C) $4x^3$ (D) $-x^4$ (E) x^4

2. If $\int_0^k \frac{x}{x^2+4} dx = \frac{1}{2}\ln 4$, where $k > 0$, then $k =$

   (A) 0 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{12}$ (E) $\frac{1}{2}\tan\left(\ln\sqrt{2}\right)$

3. A particle moves along the y-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t\cos t$. At time $t = 0$, the position of the particle is $y = 3$. Write an expression for the position $y(t)$ of the particle.

4. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$. It is known that its initial position is $x(0) = 7$.

   a. Is the particle moving to the left or to the right at time $t = 2$? Justify your answer.

   b. Is the velocity of the particle increasing or decreasing at time $t = 2$? Justify your answer.

   c. Is the speed of the particle increasing or decreasing at time $t = 2$? Justify your answer.

   d. Find the times at which the particle changes directions on the interval $0 \leq t \leq 4$. Justify your answer.

   e. Find all times on the interval $0 \leq t \leq 4$ where the speed is equal to 3.

   f. Find $x(4)$.

   g. Find the distance traveled by the particle on the interval $0 \leq t \leq 4$.

Name _____

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Calculus BC – Chapter I Sample Test (no calculators)

Show all work for free-response questions.

1. $\int_0^{\frac{\pi}{4}} e^{\tan x} \sec^2 x \, dx =$

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$

2. $\int x^7 \ln x \, dx =$

(A) $x^8 \ln x - \frac{1}{8}x^8 + C$

(B) $\frac{1}{64}x^8 \ln x - \frac{1}{64}x^8 + C$

(C) $\frac{1}{8}x^7 + \frac{1}{x} + C$

(D) $\frac{1}{8}x^8 \ln x - \frac{1}{64}x^8 + C$

3. $\int_0^1 x\sqrt{1+8x^2}\, dx =$

(A) $\frac{1}{24}$ (B) $\frac{13}{12}$ (C) $\frac{9}{8}$ (D) $\frac{52}{3}$ (E) 18

4. Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x(x^2-3)^5 \, dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5 \, du$ (B) $\int_{-2}^{13} u^5 \, du$ (C) $\frac{1}{2}\int_{-2}^{13} u^5 \, du$

(D) $\int_{-1}^{4} u^5 \, du$ (E) $\frac{1}{2}\int_{-1}^{4} u^5 \, du$

5. The position of a particle satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with the initial condition $x(0) = 4$. Find $x(12)$.

6. Let R be the region in the first quadrant under the graph $y = \frac{x}{x^2+2}$ for $0 \leq x \leq \sqrt{6}$. Find the area of R.

7. $\int (x-1)\cos(x^2-2x)\,dx =$