# Calculators Allowed

1. E
2. D
3. $y(t) = t \sin t + \cos t + 2$

4. a. left; $v(2) < 0$

   b. inc.; $a(2) > 0$

   c. dec.; $v(2)$ and $a(2)$ diff. signs

   d. 2.507, 3.545

   e. 2.842, 3.321, 3.734

   f. 4.947

   g. 7.367

# No Calculators

1. C
2. D
3. B
4. C
5. 8
6. ln 2
7. $\frac{1}{2}\sin(x^2 - 2x) + C$

Name ________________

Period ________________

Calculus BC – Chapter I Sample Test (calculators allowed)

Show all work for free-response questions.

1. Let $f$ be a differentiable function such that $\int f(x)\sin x\,dx = -f(x)\cos x + \boxed{\int 4x^3 \cos x\,dx}$.

   Which of the following could be $f(x)$?

   (A) $\cos x$ (B) $\sin x$ (C) $4x^3$ (D) $-x^4$ (E) $x^4$

$f'(x) = 4x^3$

$$\boxed{\begin{array}{ll} u = f(x) & dv = \sin x\,dx \\ du = f'(x)dx & v = -\cos x \end{array}}$$

$$\int f(x)\sin x\,dx = -f(x)\cos x - \int -\cos x\, f'(x)\,dx$$
$$= -f(x)\cos x + \boxed{\int f'(x)\cos x\,dx}$$

2. If $\int_0^k \frac{x}{x^2+4}dx = \frac{1}{2}\ln 4$, where $k > 0$, then $k =$

   (A) 0 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{12}$ (E) $\frac{1}{2}\tan\left(\ln\sqrt{2}\right)$

$$\int_0^k \frac{x}{x^2+4}dx = \int_4^{**} \frac{1}{u}\cdot\frac{1}{2}du = \frac{1}{2}\ln|u|\Big|_4^{**} = \frac{1}{2}\ln\left|x^2+4\right|\Big|_0^k$$

$$\boxed{\begin{array}{l} u = x^2+4 \\ du = 2x\,dx \\ \frac{1}{2}du = x\,dx \end{array}}$$

$$= \frac{1}{2}\ln(k^2+4) - \frac{1}{2}\ln 4 = \frac{1}{2}\ln 4$$
$$\frac{1}{2}\ln(k^2+4) = \ln 4$$
$$\ln(k^2+4) = 2\ln 4$$
$$\ln(k^2+4) = \ln 4^2$$

$k^2+4 = 16$
$k^2 = 12$
$k = \sqrt{12}$

3. A particle moves along the $y$-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t\cos t$. At time $t = 0$, the position of the particle is $y = 3$. Write an expression for the position $y(t)$ of the particle.

$$y(t) = \int t\cos t\,dt = t\sin t - \int \sin t\,dt = t\sin t + \cos t + C$$

$$\boxed{\begin{array}{ll} u = t & dv = \cos t\,dt \\ du = dt & v = \sin t \end{array}}$$

$y(t) = t\sin t + \cos t + 2$

$y(0) = 0\sin 0 + \cos 0 + C = 3$
$1 + C = 3$
$C = 2$

4. A particle moves along the $x$-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$. It is know that its initial position is $x(0) = \cancel{4}.7$

a. Is the particle moving to the left of to the right at time $t = 2$? Justify your answer.

$v(2) = -2.728 \longrightarrow$ moving left because $v(2) < 0$

b. Is the velocity of the particle increasing or decreasing at time $t = 2$? Justify your answer.

$a(2) \approx 1.588$

inc. because $a(2) > 0$

c. Is the speed of the particle increasing or decreasing at time $t = 2$? Justify your answer.

dec, because $a(2)$ and $v(2)$ are different signs

d. Find the times at which the particle changes directions on the interval $0 \leq t \leq 4$. Justify your answer.

$v(t) = 0$

$t = 2.507,\ 3.545$

$v(t)$ changes signs

e. Find all times on the interval $0 \leq t \leq 4$ where the speed is equal to 3.

$|v(t)| = 3$

$t = 2.842,\ 3.321,\ 3.734$

f. Find $x(4)$. $= x(0) + \int_0^4 v(t)\,dt = 4.947$

g. Find the distance traveled by the particle on the interval $0 \leq t \leq 4$.

$\int_0^4 |v(t)|\,dt = 7.367$

Name ____________________

Period ____________________

# Calculus BC – Chapter I Sample Test (no calculators)

Show all work for free-response questions.

1. $\int_0^{\frac{\pi}{4}} e^{\tan x} \sec^2 x dx = \int_{*}^{**} e^u du = e^u \Big|_{*}^{**}$

$\boxed{\begin{array}{l} u = \tan x \\ du = \sec^2 x\, dx \end{array}}$

(A) 0 (B) 1 (C) $e-1$ (D) $e$ (E) $e+1$

$= e^{\tan x}\Big|_0^{\pi/4} = e^{\tan\frac{\pi}{4}} - e^{\tan 0} = e^1 - e^0$

2. $\int x^7 \ln x \, dx =$

$\boxed{\begin{array}{ll} u = \ln x & dv = x^7 dx \\ du = \frac{1}{x} dx & v = \frac{1}{8}x^8 \end{array}}$

(A) $x^8 \ln x - \frac{1}{8}x^8 + C$

(B) $\frac{1}{64}x^8 \ln x - \frac{1}{64}x^8 + C$

(C) $\frac{1}{8}x^7 + \frac{1}{x} + C$

(D) $\frac{1}{8}x^8 \ln x - \frac{1}{64}x^8 + C$

$= \frac{1}{8}x^8 \ln x - \int \frac{1}{8}x^8 \cdot \frac{1}{x} dx$

$= \frac{1}{8}x^8 \ln x - \int \frac{1}{8}x^7 dx$

$= \frac{1}{8}x^8 \ln x - \frac{1}{64}x^8 + C$

3. $\int_0^1 x\sqrt{1+8x^2}\, dx = \int_{*}^{**} u^{1/2} \cdot \frac{1}{16} du = \frac{1}{16} \cdot \frac{2}{3} u^{3/2} \Big|_{*}^{**} = \frac{1}{24}(1+8x^2)^{3/2}\Big|_0^1$

$= \frac{1}{24}\left(9^{3/2} - 1^{3/2}\right)$

$= \frac{1}{24}(27-1)$

$= \frac{26}{24}$

(A) $\frac{1}{24}$ (B) $\frac{13}{12}$ (C) $\frac{9}{8}$ (D) $\frac{52}{3}$ (E) 18

$\boxed{\begin{array}{l} u = 1 + 8x^2 \\ du = 16x\, dx \\ \frac{1}{16} du = x\, dx \end{array}}$

4. Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x(x^2-3)^5\, dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5\, du$ (B) $\int_{-2}^{13} u^5\, du$ (C) $\frac{1}{2}\int_{-2}^{13} u^5\, du$

(D) $\int_{-1}^{4} u^5\, du$ (E) $\frac{1}{2}\int_{-1}^{4} u^5\, du$

$\boxed{\begin{array}{l} u = x^2 - 3 \\ du = 2x\, dx \\ \frac{1}{2} du = x\, dx \end{array}}$

$x = 4 \to u = 13$

$x = -1 \to u = -2$

$\int_{-1}^{4} x(x^2-3)^5 dx = \int_{-2}^{13} u^5 \cdot \frac{1}{2} du$

5. The position of a particle satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with the initial condition $x(0)=4$. Find $x(12)$.

$x(12) = x(0) + \int_0^{12} (2t+1)^{-1/2}\,dt = 4 + \int_{*}^{**} u^{-1/2}\,\frac{1}{2}du$

$\boxed{u = 2t+1 \quad du = 2dt \quad \frac{1}{2}du = dt}$

$= 4 + \frac{1}{2}\cdot 2u^{1/2}\Big|_{*}^{**} = 4 + \sqrt{2t+1}\Big|_0^{12}$

$= 4 + \sqrt{2(12)+1} - \sqrt{2(0)+1} = 4 + \sqrt{25} - \sqrt{1} = \boxed{8}$

6. Let $R$ be the region in the first quadrant under the graph $y = \frac{x}{x^2+2}$ for $0 \leq x \leq \sqrt{6}$.

a) Find the area of $R$.

$\int_0^{\sqrt{6}} \frac{x}{x^2+2}\,dx = \int_{*}^{**} \frac{1}{u}\cdot\frac{1}{2}du = \frac{1}{2}\ln|u|\Big|_{*}^{**}$

$\boxed{u = x^2+2 \quad du = 2x\,dx \quad \frac{1}{2}du = x\,dx}$

$= \frac{1}{2}\ln(x^2+2)\Big|_0^{\sqrt{6}} = \frac{1}{2}\ln 8 - \frac{1}{2}\ln 2$

$= \frac{1}{2}\left(\ln\left(\frac{8}{2}\right)\right) = \frac{1}{2}\ln 4 = \ln 4^{1/2}$

$= \ln 2$

7. $\int (x-1)\cos(x^2-2x)\,dx =$ $\int \cos u \cdot \frac{1}{2}du = \frac{1}{2}\sin u + C$

$\boxed{u = x^2 - 2x \quad du = (2x-2)dx \quad \frac{1}{2}du = (x-1)dx}$

$= \frac{1}{2}\sin(x^2-2x) + C$