Name ___________________

Period ___________________

Calculus BC– Chapter 9 Sample Test (no calculators)

Show all work for free-response questions.

1. Which of the following series converge conditionally?

I. $\sum_{n=1}^{\infty}(-1)^n \frac{\ln n}{n}$ II. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ III. $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n^2}$

(A) I only (B) II only (C) III only

(D) I and II only (E) I, II, and III

2. A function f has Maclaurin series given by $\frac{x^4}{2!}+\frac{x^5}{3!}+\frac{x^6}{4!}+\cdots+\frac{x^{n+2}}{n!}+\cdots$. Which of the following is an expression for $f(x)$?

(A) $-3x\sin x+3x^2$ (B) $-\cos(x^2)+1$ (C) $-x^2\cos x+x^2$

(D) $x^2e^x-x^3-x^2$ (E) $e^{x^2}-x^2-1$

3. What are all values of p for which the infinite series $\sum_{n=1}^{\infty}\frac{n}{n^p+1}$ converges?

(A) $p>0$ (B) $p\ge 1$ (C) $p>1$ (D) $p\ge 2$ (E) $p>2$

4. Suppose $\lim_{n\to\infty} a_n = \infty$ and $a_{n+1} \geq a_n > 0$ for all $n \geq 1$. Which of the following must be true?

   (A) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges

   (B) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges

   (C) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges

   (D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{a_n}$ converges

5. Which of the following series converges for all real numbers x?

   (A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$ (B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ (C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

   (D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$ (E) $\sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$

6. Which of the following statements about $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$ is true?

   (A) The series can be shown to diverge by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

   (B) The series can be shown to diverge by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

   (C) The series can be shown to converge by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

   (D) The series can be shown to converge by the Alternating Series Test

7. What is the sum of the series $1 + \ln 2 + \frac{(\ln 2)^2}{2!} + \cdots + \frac{(\ln 2)^n}{n!} + \cdots$?

   (A) $\ln 2$ (B) $\ln(1 + \ln 2)$ (C) 2

   (D) e^2 (E) The series diverges

Calculus BC– Chapter 9 Sample Test (no calculators)

8. The sum of the infinite geometric series $\frac{3}{2}+\frac{9}{16}+\frac{27}{128}+\frac{81}{1{,}024}+\cdots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

9. Let f be the function given by $f(x)=\ln(3-x)$. The third-degree Taylor polynomial for f about $x=2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$
(B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$
(C) $(x-2)+(x-2)^2+(x-2)^3$
(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$
(E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

10. If f is a function such that $f'(x)=e^{x^3}$, find the coefficient of x^7 in the Maclaruin series for $f(x)$.

11. The Maclaurin series for the function f is given by $f(x)=\sum_{n=2}^{\infty}\frac{(-1)^n(2x)^n}{n-1}$ on its interval of convergence. Find the interval of convergence for the Maclaurin series of f. Justify your answer.

12. The function f is defined by the power series

$$f(x) = 1 + (x+1) + (x+1)^2 + \cdots + (x+1)^n + \cdots = \sum_{n=0}^{\infty}(x+1)^n$$

for all real numbers x for which the series converges.

a) Find the interval of convergence of the power series for f. Justify your answer.

b) The power series above is the Taylor series for f about $x = -1$. Find the sum of the series for f.

c) Let h be the function defined by $h(x) = f(x^2 - 1)$. Find the first three nonzero terms and the general term of the Taylor series for h about $x = 0$, and find the value of $h\left(\frac{1}{2}\right)$.

Calculus BC– Chapter 9 Sample Test (no calculators)

$$f(x) = \begin{cases} \dfrac{\cos x - 1}{x^2} & \text{for } x \neq 0 \\ -\dfrac{1}{2} & \text{for } x = 0 \end{cases}$$

13. The function f, defined above, has derivatives of all orders. Let g be the function defined by $g(x) = 1 + \int_0^x f(t)\,dt$.

a) Write the first three nonzero terms and the general term of the Taylor series for $\cos x$ about $x = 0$. Use this series to write the first three nonzero terms and the general term of the Taylor series for f about $x = 0$.

b) Use the Taylor series for f about $x = 0$ found in part (a) to determine whether f has a relative maximum, relative minimum, or neither at $x = 0$. Give a reason for your answer.

c) Write the fifth-degree Taylor polynomial for g about $x = 0$.

d) The Taylor series for g about $x = 0$, evaluated at $x = 1$, is an alternating series with individual terms that decrease in absolute value to 0. Use the third-degree Taylor polynomial for g about $x = 0$ to estimate the value of $g(1)$. Explain why this estimate differs from the actual value of $g(1)$ by less than $\frac{1}{6!}$

14. Let $f(x) = \ln(1 + x^3)$.

a) The Maclaurin series for $\ln(1 + x)$ is $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots$. Use the series to write the first four nonzero terms and the general term of the Maclaurin series for f.

b) The radius of convergence of the Maclaurin series for f is 1. Determine the interval of convergence. Show the work that leads to your answer.

c) Write the first four nonzero terms of the Maclaurin series for $f'(t^2)$. If $g(x) = \int_0^x f'(t^2)\,dt$, use the first two nonzero terms of the Maclaurin series for g to approximate $g(1)$.

d) The Maclaurin series for g, evaluated at $x = 1$, is a convergent alternating series with individual terms that decrease in absolute value to 0. Show that your approximation in (c) must differ from $g(1)$ by less than $\frac{1}{5}$.