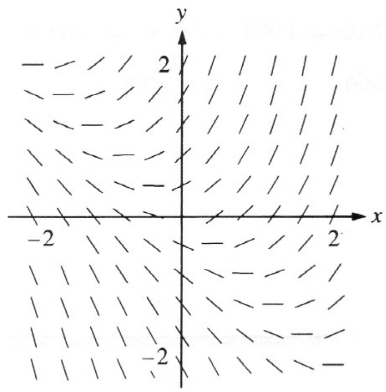
Name ___________________

Period ___________________

Calculus BC – Chapter 8 Sample Test (no calculators)

Show all work for free-response questions.



1. Shown above is a slope field for which of the following differential equations?

   (A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$

   (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

2. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?

   (A) 3 (B) 5 (C) 6 (D) 10 (E) 12

3. The temperature of a solid at time $t \geq 0$ is modeled by the nonconstant function H and increases according to the differential equation $\frac{dH}{dt} = 2H + 1$, where $H(t)$ is measured in degrees Fahrenheit and t is measured in hours. Which of the following much be true?

   (A) $H = H^2 + t + C$

   (B) $\ln|2H + 1| = \frac{t}{2} + C$

   (C) $\ln|2H + 1| = t + C$

   (D) $\ln|2H + 1| = 2t + C$

4. The population $P(t)$ of a species satisfies the logistic differential equation $\dfrac{dP}{dt} = P\left(2 - \dfrac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim\limits_{t\to\infty} P(t)$?

   (A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000

5. The equation $y = 2e^{6x} - 5$ is a particular solution to which of the following differential equations?

   (A) $y' - 6y - 30 = 0$

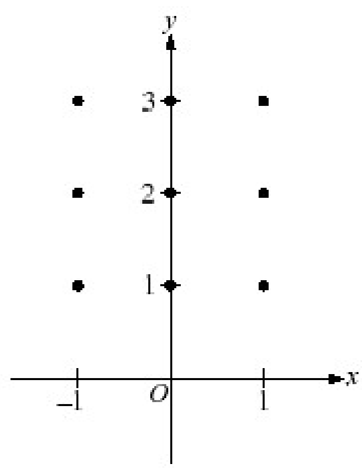
   (B) $2y' - 12y + 5 = 0$

   (C) $y'' - 5y' - 6y = 0$

   (D) $y'' - 2y' + y + 5 = 0$

6. Consider the differential equation given by $\dfrac{dy}{dx} = \dfrac{xy}{2}$.

   a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



   b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $y(0) = 3$.

7. In a national park, the population of mountain lions grows over time. At time $t = 0$, where t is measured in years, the population is found to be 20 mountain lions.

   a) One zoologist suggests a population model P that satisfies the differential equation $\frac{dP}{dt} = \frac{1}{4}(220 - P)$. Use separation of variables to solve this differential equation for P with the initial condition $P(0) = 20$.

   b) A second zoologist suggests a population model Q that satisfies $\frac{dQ}{dt} = \frac{1}{500}Q(220 - Q)$. Find the value of $\frac{dQ}{dt}$ at the time when Q grows most rapidly.

   c) For the population model Q introduced in part (b), use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $Q(10)$. Show the computations that lead to your answer.

8. Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ particular solution to this differential equation with the initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of x.

a) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

b) Find $\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.

c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with the initial condition $f(1) = 0$.