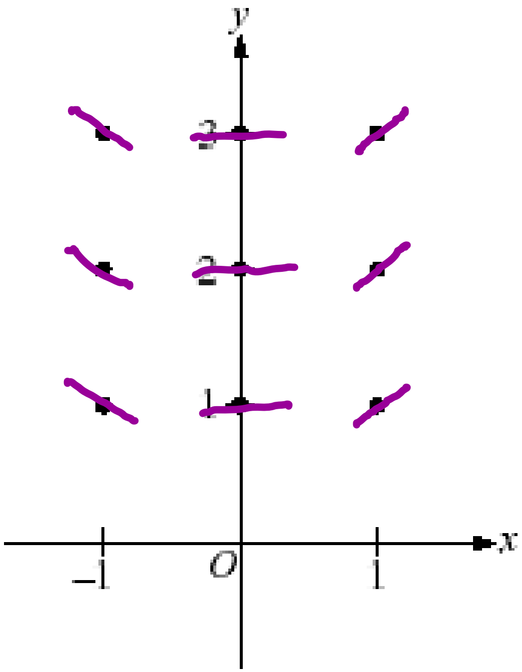
# No Calculators

1. C 2. C 3. D 4. E 5. A

6. a. → b. $y = 3e^{x^2/4}$




7. a. $P = 220 - 200e^{-t/4}$ b. $\frac{121}{5}$ c. 156

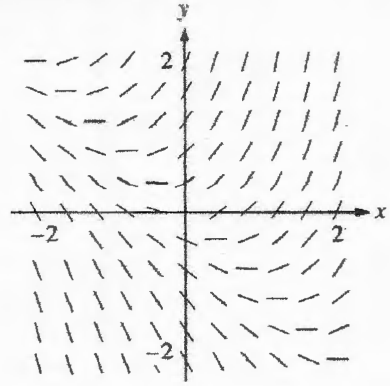
8. a. -1.25 b. $\frac{1}{3}$ c. $y = 1 - e^{1-x}$

Name ________________

Period ________________

Calculus BC – Chapter 8 Sample Test (no calculators)

Show all work for free-response questions.




1. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$

(D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

2. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?

(A) 3 (B) 5 (C) 6 (D) 10 (E) 12

| $\Delta x$ | $x_1$ | $y_1$ | m | $y_2$ |
|---|---|---|---|---|
| .5 | 1 | 2 | 1+2=3 | 2+.5(3)=3.5 |
| .5 | 1.5 | 3.5 | 1.5+3.5=5 | 3.5+.5(5)=6 |
| .5 | 2 | 6 | | |

3. The temperature of a solid at time $t \geq 0$ is modeled by the nonconstant function $H$ and increases according to the differential equation $\frac{dH}{dt} = 2H + 1$, where $H(t)$ is measured in degrees Fahrenheit and $t$ is measured in hours. Which of the following much be true?

$\frac{1}{2H+1}\, dH = dt$

$\frac{1}{2} \ln|2H+1| = t + C$

$\ln|2H+1| = 2t + D$

(A) $H = H^2 + t + C$

(B) $\ln|2H + 1| = \frac{t}{2} + C$

(C) $\ln|2H + 1| = t + C$

(D) $\ln|2H + 1| = 2t + C$

# Calculus BC -- Chapter 8 Sample Test (no calculators)

4. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and $t$ is the time in years. What is $\lim_{t \to \infty} P(t)$?

$2 - \frac{P}{5000} = 0$

$P = 10000$

(A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000

5. The equation $y = 2e^{6x} - 5$ is a particular solution to which of the following differential equations?

$\longrightarrow y' = 12e^{6x} \rightarrow y'' = 72e^{6x}$

(A) $y' - 6y - 30 = 0$ $\quad 12e^{6t} - 6(2e^{6t} - 5) - 30 = 0$ ✓

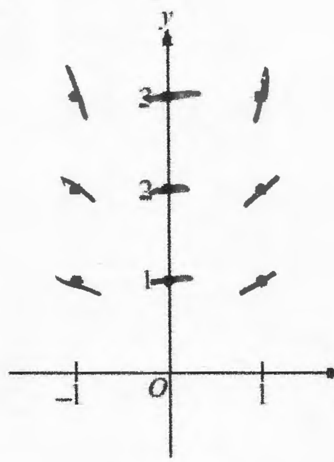
(B) $2y' - 12y + 5 = 0$ $\quad 2(12e^{6t}) - 12(2e^{6x} - 5) + 5 = 0$ ✗

(C) $y'' - 5y' - 6y = 0$ $\quad 72e^{6t} - 5(12e^{6t}) - 6(2e^{6t} - 5) = 0$ ✗

(D) $y'' - 2y' + y + 5 = 0$ $\quad 72e^{6t} - 2(12e^{6t}) + 2e^{6t} - 5 + 5 = 0$ ✗

6. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.




b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $y(0) = 3$.

$\frac{dy}{dx} = \frac{xy}{2}$

$\frac{1}{y}\,dy = \frac{x}{2}\,dx$

$\ln|y| = \frac{1}{4}x^2 + C$

$y = De^{\frac{1}{4}x^2}$

$\rightarrow 3 = De^0$

$D = 3$

$y = 3e^{\frac{1}{4}x^2}$

<u>Calculus BC -- Chapter 8 Sample Test (no calculators)</u>

7. In a national park, the population of mountain lions grows over time. At time $t = 0$, where $t$ is measured in years, the population is found to be 20 mountain lions.

a) One zoologist suggests a population model $P$ that satisfies the differential equation $\frac{dP}{dt} = \frac{1}{4}(220 - P)$. Use separation of variables to solve this differential equation for $P$ with the initial condition $P(0) = 20$.

$$\frac{1}{220-P}\,dP = \frac{1}{4}\,dt$$

$$-\ln|220-P| = \frac{1}{4}t + C$$

$$\ln|220-P| = -\frac{1}{4}t + D$$

$$220 - P = Fe^{-1/4 t}$$

$$P = 220 + Ge^{-\frac{1}{4}t}$$

$$20 = 220 + Ge^{0}$$

$$G = -200$$

$$P = 220 - 200e^{-\frac{1}{4}t}$$

b) A second zoologist suggests a population model $Q$ that satisfies $\frac{dQ}{dt} = \frac{1}{500}Q(220 - Q)$. Find the value of $\frac{dQ}{dt}$ at the time when $Q$ grows most rapidly.

Carrying capacity $\Rightarrow Q = 220$

Fastist Growth $\Rightarrow Q = 110$

$$\frac{dQ}{dt} = \frac{1}{500}(110)(220-110) = \frac{121}{5}$$

c) For the population model $Q$ introduced in part (b), use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $Q(10)$. Show the computations that lead to your answer.

| $\Delta t$ | $t_1$ | $Q_1$ | $m$ | $Q_2$ |
|---|---|---|---|---|
| 5 | 0 | 20 | $\frac{1}{500}(20)(220-20) = 8$ | $20 + 5(8) = 60$ |
| 5 | 5 | 60 | $\frac{1}{500}(60)(220-60) = \frac{96}{5}$ | $60 + 5\left(\frac{96}{5}\right) = 156$ |
| 5 | 10 | 156 | | |

$$Q(10) \approx 156$$

Calculus BC -- Chapter 8 Sample Test (no calculators)

8. Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ particular solution to this differential equation with the initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of $x$.

a) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

| $\Delta x$ | $x_1$ | $y_1$ | $m$ | $y_2$ |
|---|---|---|---|---|
| $-.5$ | $1$ | $0$ | $1-0=1$ | $0+(-.5)(1)=-.5$ |
| $-.5$ | $.5$ | $-.5$ | $1-(-.5)=1.5$ | $-.5+(-.5)(1.5)=-5/4$ |
| $-.5$ | $0$ | $-5/4$ | | |

$f(0) \approx -\frac{5}{4}$

b) Find $\lim\limits_{x \to 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.

$$\lim_{x\to 1} \frac{\overset{\to 0}{f(x)}}{\underset{\to 0}{x^3-1}} \overset{L}{=} \lim_{x\to 1} \frac{\overset{\to 1-0}{f'(x)}}{\underset{\to 3}{3x^2}} = \frac{1}{3}$$

c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with the initial condition $f(1) = 0$.

$$\frac{dy}{dx} = 1 - y$$
$$\frac{1}{1-y}\,dy = dx$$
$$-\ln|1-y| = x + C$$
$$\ln|1-y| = -x + D$$
$$1 - y = Fe^{-x}$$
$$y = 1 + Ge^{-x}$$

$$0 = 1 + Ge^{-1}$$
$$-1 = Ge^{-1}$$
$$G = -e$$

$$y = 1 - e \cdot e^{-x}$$
$$\boxed{y = 1 - e^{1-x}}$$