1. C 2. D 3. C 4. A 5. C 6. A

7. $\frac{3}{2}x^2 - 5x + 4\ln|x-2| + \frac{7}{x-2} + c$

8. $3\ln\left|\frac{3}{x} - \frac{\sqrt{9-x^2}}{x}\right| + \sqrt{9-x^2} + c$ or $-3\ln\left|\frac{3}{x} + \frac{\sqrt{9-x^2}}{x}\right| + \sqrt{9-x^2} + c$

9. $\frac{1}{7}\tan^7 x + \frac{1}{5}\tan^5 x + c$

10. $\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right) + \frac{x\sqrt{9-4x^2}}{2} + c$

11. $-\frac{1}{5}\cos^5 t + \frac{2}{3}\cos^3 t - \cos t + c$

12. $\frac{1}{3}\left(4+x^2\right)^{3/2} - 4\sqrt{4+x^2} + c$

13. $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + c$

14. $\frac{1}{2}x^2 - 5x + 25\ln|x+5| + c$

or

$\frac{1}{2}(x+5)^2 - 10x + 25\ln|x+5| + c$

15. $2\ln|2x-1| + 3\ln|x+1| + c$

Name ____________________

Period ____________________

Calculus BC – Chapter 7 Sample Test (no calculators)

Show all work for free-response questions.

1. What are all values of p for which $\int_1^\infty \frac{1}{x^{2p}}\,dx$ converges?

$= \lim_{R\to\infty} \int_1^R x^{-2p}\,dx = \lim_{R\to\infty} \frac{1}{-2p+1} x^{-2p+1}\Big|_1^R$

$= \lim_{R\to\infty} \frac{1}{-2p+1}\frac{1}{x^{2p-1}} - \frac{1}{-2p+1}$

conv. if $2p-1>0$, $p>\frac{1}{2}$

(A) $p<\frac{1}{2}$ (B) $p>0$ (C) $p>\frac{1}{2}$

(D) $p>1$ (E) There are no values of p

2. $\int \frac{2x}{(x+2)(x+1)}\,dx = \int \frac{4}{x+2} + \frac{-2}{x+1}\,dx = 4\ln|x+2| - 2\ln|x+1| + C$

$\frac{2x}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$

$2x = A(x+1) + B(x+2)$

$x=-1$: $-2 = B(1) \to B=-2$

$x=-2$: $-4 = A(-1) \to A=4$

(A) $\ln|x+2| + \ln|x+1| + C$

(B) $\ln|x+2| + \ln|x+1| - 3x + C$

(C) $-4\ln|x+2| + 2\ln|x+1| + C$

(D) $4\ln|x+2| - 2\ln|x+1| + C$

(E) $2\ln|x| + \frac{2}{3}x + \frac{1}{2}x^2 + C$

3. $\int_0^\infty x^2 e^{-x^3}\,dx$ is $= \lim_{R\to\infty}\int_0^R x^2 e^{-x^3}\,dx = \lim_{R\to\infty} -\frac{1}{3}e^{-x^3}\Big|_0^R = \lim_{R\to\infty} \frac{-1}{3e^{R^3}} + \frac{1}{3}$

$\frac{-1}{3e^{R^3}} \to 0$

(A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$

(D) 1 (E) divergent

$\int x^2 e^{-x^3}\,dx = -\frac{1}{3}\int e^u\,du = -\frac{1}{3}e^{-x^3}$

$u=-x^3$

$du = -3x^2\,dx$

Calculus BC -- Chapter 7 Sample Test (no calculators)

4. $\int \frac{dx}{(x-1)(x+3)} =$ $\int \frac{1/4}{x-1} + \frac{-1/4}{x+3}\,dx = \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+3| + C$
$= \frac{1}{4}\left[\ln|x-1| - \ln|x+3|\right] + C$

(A) $\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$

(B) $\frac{1}{4}\ln\left|\frac{x+3}{x-1}\right| + C$

(C) $\frac{1}{2}\ln|(x-1)(x+3)| + C$

(D) $\frac{1}{2}\ln\left|\frac{2x+2}{(x-1)(x+3)}\right| + C$

(E) $\ln|(x-1)(x+3)| + C$

$$\boxed{\begin{aligned} \frac{1}{(x-1)(x+3)} &= \frac{A}{x-1} + \frac{B}{x+3} \\ 1 &= A(x+3) + B(x-1) \\ x=1:\ 1 &= A(4) \to A = \tfrac{1}{4} \\ x=-3:\ 1 &= B(-4) \to B = -\tfrac{1}{4} \end{aligned}}$$

$$\boxed{\begin{aligned} u &= x^2-4 \\ du &= 2x\,dx \\ \tfrac{1}{2}du &= x\,dx \end{aligned}}$$

5. $\int \frac{x}{x^2-4}\,dx =$ $\int \frac{1}{u}\cdot\frac{1}{2}du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2-4| + C$

(A) $\frac{-1}{4(x^2-4)^2} + C$

(B) $\frac{1}{2(x^2-4)} + C$

(C) $\frac{1}{2}\ln|x^2-4| + C$

(D) $2\ln|x^2-4| + C$

(E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

Calculus BC -- Chapter 7 Sample Test (no calculators)

6. $\int \sec^5 x \tan^3 x \, dx =$ $\int \sec^4 x \tan^2 x \sec x \tan x \, dx$

$= \int \sec^4 x (\sec^2 x - 1) \sec x \tan x \, dx$

$\boxed{\begin{array}{l} u = \sec x \\ du = \sec x \tan x \, dx \end{array}}$

(A) $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$

(B) $\frac{1}{24}\sec^6 x \tan^4 x + C$

(C) $\frac{1}{4}\sec^4 x \tan x + C$

(D) $\frac{1}{8}\sec^8 x - \frac{1}{6}\sec^6 x + C$

(E) $\frac{1}{5}\sec^5 x - \frac{1}{7}\sec^7 x + C$

$= \int u^4(u^2 - 1)\, du$

$= \int u^6 - u^4 \, du$

$= \frac{1}{7}u^7 - \frac{1}{5}u^5 + C$

$= \frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$

7. $\int \frac{3x^3 - 17x^2 + 36x - 35}{x^2 - 4x + 4} dx$ $= \int 3x - 5 + \frac{4x - 15}{(x-2)^2} dx = \int 3x - 5 + \frac{4}{x-2} + \frac{-7}{(x-2)^2} dx$

$= \frac{3}{2}x^2 - 5x + 4\ln|x-2| + \frac{7}{x-2} + C$

$$\begin{array}{r} 3x - 5 + \frac{4x-15}{x^2-4x+4} \\ x^2 - 4x + 4 \overline{\big)\, 3x^3 - 17x^2 + 36x - 35} \\ -(3x^3 - 12x^2 + 12x) \\ \hline -5x^2 + 24x - 35 \\ -(-5x^2 + 20x - 20) \\ \hline 4x - 15 \end{array}$$

$\frac{4x-15}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$

$4x - 15 = A(x-2) + B$

$x = 2: \quad -7 = B$

$x = 0: \quad -15 = A(-2) - 7 \rightarrow A = 4$

8. $\int \frac{\sqrt{9 - x^2}}{x} dx$

$\boxed{\begin{array}{l} x = 3\sin\theta \\ dx = 3\cos\theta \, d\theta \end{array}}$

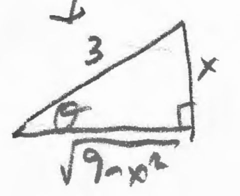
$= \int \frac{\sqrt{9 - 9\sin^2\theta}}{3\sin\theta} 3\cos\theta \, d\theta = \int \frac{\sqrt{9\cos^2\theta}}{3\sin\theta} 3\cos\theta \, d\theta$

$\sin\theta = \frac{x}{3}$

$= \int \frac{3\cos^2\theta}{\sin\theta} d\theta = \int 3\frac{(1 - \sin^2\theta)}{\sin\theta} d\theta = \int 3(\csc\theta - \sin\theta) d\theta$

$= 3\left[-\ln|\csc\theta + \cot\theta| + \cos\theta\right] + C$

$= 3\left[-\ln\left|\frac{3}{x} + \frac{\sqrt{9-x^2}}{x}\right| + \frac{\sqrt{9-x^2}}{3}\right] + C$



Calculus BC -- Chapter 7 Sample Test (no calculators)

9. $\int \tan^4 x \sec^4 x\,dx = \int \tan^4 x \sec^2 x \sec^2 x\,dx = \int \tan^4 x(\tan^2 x+1)\sec^2 x\,dx$

$$\boxed{\begin{array}{l} u=\tan x \\ du=\sec^2 x\,dx \end{array}}$$

$= \int u^4(u^2+1)\,du = \int u^6+u^4\,du$

$= \frac{1}{7}u^7+\frac{1}{5}u^5+C = \frac{1}{7}\tan^7 x+\frac{1}{5}\tan^5 x+C$

10. $\int \sqrt{9-4x^2}\,dx = \int \sqrt{9-9\sin^2\theta}\cdot\frac{3}{2}\cos\theta\,d\theta = \int\sqrt{9\cos^2\theta}\cdot\frac{3}{2}\cos\theta\,d\theta = \int \frac{9}{2}\cos^2\theta\,d\theta$

$= \int \frac{9}{4}(1+\cos 2\theta)\,d\theta = \frac{9}{4}\left(\theta+\frac{1}{2}\sin 2\theta\right)+C$

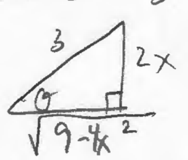
$= \frac{9}{4}\left(\theta+\sin\theta\cos\theta\right)+C = \frac{9}{4}\left(\sin^{-1}\left(\frac{2x}{3}\right)+\frac{2x}{3}\cdot\frac{\sqrt{9-4x^2}}{3}\right)+C$

$= \frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)+\frac{x}{2}\sqrt{9-4x^2}+C$

$$\boxed{\begin{array}{l} 9-4x^2 \\ 9-9\sin^2\theta \\ 9(1-\sin^2\theta) \end{array}}$$

$$\boxed{\begin{array}{l} 4x^2=9\sin^2\theta \\ 2x=3\sin\theta \\ x=\frac{3}{2}\sin\theta \\ dx=\frac{3}{2}\cos\theta\,d\theta \end{array}}$$

$\rightarrow \sin\theta=\frac{2x}{3} \rightarrow \theta=\sin^{-1}\left(\frac{2x}{3}\right)$



11. $\int \sin^5 t\,dt = \int \sin^4 t\sin t\,dt = \int(\sin^2 t)^2\sin t\,dt = \int(1-\cos^2 t)^2\sin t\,dt$

$$\boxed{\begin{array}{l} u=\cos t \\ du=-\sin t\,dt \\ -du=\sin t\,dt \end{array}}$$

$= \int(1-u^2)^2(-1)\,du = \int -u^4+2u^2-1\,du = -\frac{1}{5}u^5+\frac{2}{3}u^3-u+C$

$= -\frac{1}{5}\cos^5 t+\frac{2}{3}\cos^3 t-\cos t+C$

12. $\int \frac{x^3}{\sqrt{4+x^2}}\,dx = \int \frac{8\tan^3\theta}{\sqrt{4+4\tan^2\theta}}2\sec^2\theta\,d\theta = \int\frac{8\tan^3\theta}{\sqrt{4\sec^2\theta}}2\sec^2\theta\,d\theta$

$= \int 8\tan^3\theta\sec\theta\,d\theta = \int 8\tan^2\theta\sec\theta\tan\theta\,d\theta$

$= \int 8(\sec^2\theta-1)\sec\theta\tan\theta\,d\theta = \int 8(u^2-1)\,du$

$= \frac{8}{3}u^3-8u+C = \frac{8}{3}\sec^3\theta-8\sec\theta+C$

$= \frac{8}{3}\cdot\left(\frac{\sqrt{4+x^2}}{2}\right)^3-8\left(\frac{\sqrt{4+x^2}}{2}\right)+C$

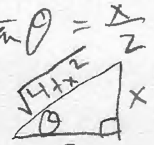
$= \frac{1}{3}(4+x^2)^{3/2}-4\sqrt{4+x^2}+C$

$$\boxed{\begin{array}{l} x=2\tan\theta \\ dx=2\sec^2\theta\,d\theta \end{array}}$$

$u=\sec\theta$

$du=\sec\theta\tan\theta\,d\theta$

$\rightarrow \tan\theta=\frac{x}{2}$



Calculus BC -- Chapter 7 Sample Test (no calculators)

13. $\int x^3 \ln x \, dx$ $= \frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^4 \cdot \frac{1}{x} dx = \frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^3 dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$

$u = \ln x \quad dv = x^3 dx$

$du = \frac{1}{x} dx \quad v = \frac{1}{4}x^4$

14. $\int \frac{x^2}{x+5} dx = \int x - 5 + \frac{25}{x+5} dx = \frac{1}{2}x^2 - 5x + 25 \ln|x+5| + C$

$$\begin{array}{r|l} & x - 5 + \frac{25}{x+5} \\ x+5 & x^2 \\ & -(x^2+5x) \\ & -5x \\ & -(-5x-25) \\ & 25 \end{array}$$

15. $\int \frac{10x+1}{(2x-1)(x+1)} dx$

$= \int \frac{4}{2x-1} + \frac{3}{x+1} dx = 2 \ln|2x-1| + 3 \ln|x+1| + C$

$\frac{10x+1}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1}$

$10x + 1 = A(x+1) + B(2x-1)$

$x = -1$: $-9 = B(-3) \rightarrow B = 3$

$x = \frac{1}{2}$: $6 = A\left(\frac{3}{2}\right) \rightarrow A = 4$