Name _____

Period _____

Calculus BC – Chapter 5 Sample Test (calculators allowed)

Show all work for free-response questions.

1. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

   (A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1640.250

2. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table below. Using subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

x	2	5	7	8
$f(x)$	10	30	40	20

   (A) 110 (B) 130 (C) 160 (D) 190 (E) 210

3. Find the derivative of the function $\int_x^{x^9} \ln t \, dt$.

   (A) $x(x^8 - 1)\ln x$

   (B) $(81x^8 - 1)\ln x$

   (C) $8\ln x$

   (D) $\frac{9}{x}$

4. $\int_0^x \sin t \, dt$

   (A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

5. Let $f(x)$ be the function that is defined for all real numbers x and that has the following properties:

   (i) $f''(x) = 24x - 18$ (ii) $f'(1) = -6$ (iii) $f(2) = 0$

   Find an expression for $f(x)$.

6. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The graph of R is concave down for all values of t on the interval. The table below shows the rate as measured every 3 hours for a 24-hour period.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

   a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$.

   b) Is the approximation an overestimate or underestimate of the exact value. Give a reason for your answer.

   c) Using correct units, explain the meaning of $\int_0^{24} R(t)dt$ in the context of this problem.

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Calculus BC – Chapter 5 Sample Test (no calculators)

Show all work for free-response questions.

1. If $f'(x) = 3x^2$ and $f(-1) = 2$, then $\int_0^2 f(x)dx =$

   (A) $\frac{8}{3}$ (B) 4 (C) 7 (D) 10 (E) 28

2. If $f(x)$ is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)dx =$

   (A) $2F(3) - 2F(1)$ (B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$ (C) $2F(6) - 2F(2)$

   (D) $F(6) - F(2)$ (E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

3. Let R be the region bounded by the graph $y = \cos x$, the x-axis, and the y-axis.
   a) Find the area of the region R.

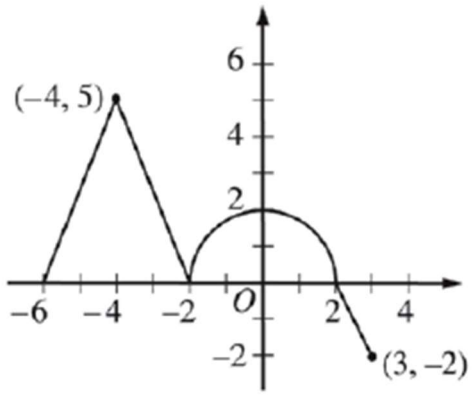
   b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

4. A particle moves along the x-axis so that its acceleration at any time x is given by $a(t) = 6t - 18$. At time $t = 0$, the velocity of the particle is $v(0) = 24$, and at time $t = 1$, its position is $x(1) = 20$.

   a) Write an expression for the velocity $v(t)$ of the particle at any time t.

   b) Write an expression for the position $x(t)$ of the particle at any time t.

   c) For what values of t is the particle at rest?



Graph of f

5. The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)dt$.

   a) Find $g(-6)$ and $g(3)$.

   b) Find $g'(0)$.

   c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent line. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

   d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.