Calculators Allowed

1. C 2. C 3. B 4. E

5. $f(x) = 4x^3 - 9x^2 + 4$

6. a. 258.6 b. over because R is CCD
 c. Amount of water that flows, in gallons, over 24 hr. period

No Calculators

1. D 2. E 3. a. 1 b. $\frac{\pi}{6}$

4. a. $v(t) = 3t^2 - 18t + 24$ b. $x(t) = t^3 - 9t^2 + 24t + 4$ c. $t = 2$, $t = 4$

5. a. $g(-6) = -10$, $g(3) = 2\pi - 1$ b. 2

 c. -2 is neither (no change in sign of f')

 2 is local max (f' goes from pos. to neg.)

 d. -4, -2, 0 (slope of f' changes signs)

Name ____________________

Period ____________________

Calculus BC – Chapter 5 Sample Test (calculators allowed)

Show all work for free-response questions.

1. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$ $F(1) + \int_1^9 \frac{(\ln x)^3}{x}dx$

   (A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1640.250

2. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table below. Using subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)dx$? $= \frac{1}{2}[f(2)+f(5)]3 + \frac{1}{2}[f(5)+f(7)]\cdot 2 + \frac{1}{2}[f(7)+f(8)]\cdot 1$

x	2	5	7	8
$f(x)$	10	30	40	20

   (A) 110 (B) 130 (C) 160 (D) 190 (E) 210

3. Find the derivative of the function $\int_x^{x^9} \ln t\, dt$. $= \ln(x^9)\cdot 9x^8 - \ln x(1)$
   $= 9x^8 \cdot 9\ln x - \ln x$
   $= (81x^8 - 1)\ln x$

   (A) $x(x^8 - 1)\ln x$

   (B) $(81x^8 - 1)\ln x$

   (C) $8\ln x$

   (D) $\frac{9}{x}$

4. $\int_0^x \sin t\, dt$ $= -\cos t\Big|_0^x = -\cos x + \cos 0$

   (A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

5. Let $f(x)$ be the function that is defined for all real numbers x and that has the following properties:

(i) $f''(x) = 24x - 18$ (ii) $f'(1) = -6$ (iii) $f(2) = 0$

Find an expression for $f(x)$.

$f'(x) = 12x^2 - 18x + C$

$f'(1) = 12 - 18 + C = -6 \rightarrow C = 0$

$f'(x) = 12x^2 - 18x$

$f(x) = 4x^3 - 9x^2 + D$

$f(2) = 32 - 36 + D = 0$

$D = 4$

$\boxed{f(x) = 4x^3 - 9x^2 + 4}$

6. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The graph of R is concave down for all values of t on the interval. The table below shows the rate as measured every 3 hours for a 24-hour period.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$. $= R(3)\cdot 6 + R(9)\cdot 6 + R(15)\cdot 6 + R(21)\cdot 6$

$= 258.6$

b) Is the approximation an overestimate or underestimate of the exact value. Give a reason for your answer.

over, because R is concave down.

c) Using correct units, explain the meaning of $\int_0^{24} R(t)dt$ in the context of this problem.

Amount of water that flows through the pipe, im gallons, over the 24 hr. period

Name ____________________

Period ____________________

Calculus BC – Chapter 5 Sample Test (no calculators)

Show all work for free-response questions.

1. If $f'(x) = 3x^2$ and $f(-1) = 2$, then $\int_0^2 f(x)dx =$ $\int_0^2 (x^3+3)dx = \frac{1}{4}x^4+3x\Big|_0^2 = 4+6$

$f(x)=x^3+C$
$f(-1)=(-1)^3+C=2$
$C=3$

(A) $\frac{8}{3}$ (B) 4 (C) 7 (D) 10 (E) 28

2. If $f(x)$ is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)dx =$ $\frac{1}{2}F(2x)\Big|_1^3 = \frac{1}{2}F(6) - \frac{1}{2}F(2)$

(A) $2F(3)-2F(1)$ (B) $\frac{1}{2}F(3)-\frac{1}{2}F(1)$ (C) $2F(6)-2F(2)$

(D) $F(6)-F(2)$ (E) $\frac{1}{2}F(6)-\frac{1}{2}F(2)$

3. Let R be the region bounded by the graph $y = \cos x$, the x-axis, and the line $x = \frac{\pi}{2}$.

a) Find the area of the region R.

$\int_0^{\pi/2} \cos x\,dx = \sin x\Big|_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0 = \boxed{1}$

b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

$\int_0^h \cos x\,dx = \sin x\Big|_0^h = \sin h - \sin 0 = \sin h = \frac{1}{2}$

$\boxed{h = \frac{\pi}{6}}$

4. A particle moves along the x-axis so that its acceleration at any time x is given by $a(t) = 6t - 18$. At time $t = 0$, the velocity of the particle is $v(0) = 24$, and at time $t = 1$, its position is $x(1) = 20$.

a) Write an expression for the velocity $v(t)$ of the particle at any time t.

$v(t) = 3t^2 - 18t + C$
$v(0) = 0 + 0 + C = 24 \rightarrow C = 24$

$v(t) = 3t^2 - 18t + 24$

b) Write an expression for the position $x(t)$ of the particle at any time t.

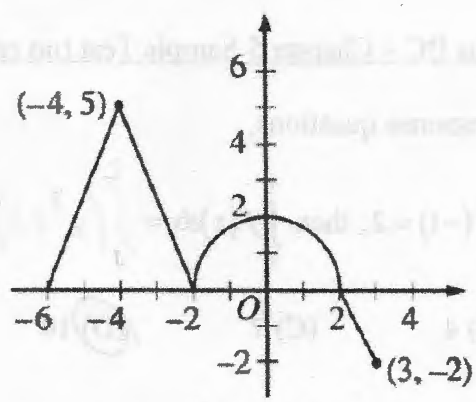
$x(t) = t^3 - 9t^2 + 24t + D$
$x(1) = 1 - 9 + 24 + D = 20$
$D = 4$

$x(t) = t^3 - 9t^2 + 24t + 4$

c) For what values of t is the particle at rest?

$v(t) = 3t^2 - 18t + 24$
$= 3(t^2 - 6t + 8)$
$= 3(t-2)(t-4) = 0$ $\rightarrow$ $\boxed{t = 2, 4}$

Calculus BC -- Chapter 5 Sample Test (no calculators)



Graph of f

5. The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.

   a) Find $g(-6)$ and $g(3)$.

   $$g(-6) = \int_{-2}^{-6} f(t)\,dt = -\int_{-6}^{-2} f(t)\,dt = -\tfrac{1}{2}(4)(5) = -10$$

   $$g(3) = \int_{-2}^{3} f(t)\,dt = \tfrac{1}{2}\pi(2)^2 - \tfrac{1}{2}(1)(2) = 2\pi - 1$$

   b) Find $g'(0)$.

   $$g'(x) = f(x)$$
   $$g'(0) = f(0) = 2$$

   c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent line. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

   $g' = 0 \rightarrow x = -2, 2$ $x = -2$, neither g' doesn't change signs

   $x = 2$, local max, g' goes pos to neg

   d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

   $g'' = f'$ $x = -4, -2, 0$

   slope of f changes signs