Review, Part ~~3~~ 2

<u>Riemann Sums</u>

$A = \frac{1}{2}h(b_1 + b_2)$

→ LHS, RHS, Midpoint, Trapezoid

→ Don't forget to write the $f(2) + f(3)$ step

SAMPLE A

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

$y = f(1) + f'(1)(x-1)$

$y = 15 + 8(x-1)$

$f(1.4) \approx 15 + 8(1.4-1) = 18.2$

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

b) $\int_1^{1.4} f'(x)\,dx = .2f'(1.1) + .2f'(1.3)$

$= .2(10) + .2(13) = 4.6$

$f(1.4) = f(1) + \int_1^{1.4} f'(x)\,dx$

$\approx 15 + 4.6 = 19.6$

c)

Δx	x_1	y_1	m	$y_2 = y_1 + m \cdot \Delta x$
.2	1	15	8	$15 + 8(.2) = 16.6$
.2	1.2	16.6	12	$16.6 + 12(.2) = 19$
.2	1.4	19		

$f(1.4) \approx 19$

d) $f(x) \approx f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2$

$\approx 15 + 8(x-1) + \frac{20}{2}(x-1)^2$

$f(1.4) \approx 15 + 8(1.4-1) + 10(1.4-1)^2$

≈ 19.8

<u>Fundamental Theorem of Calculus</u>

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

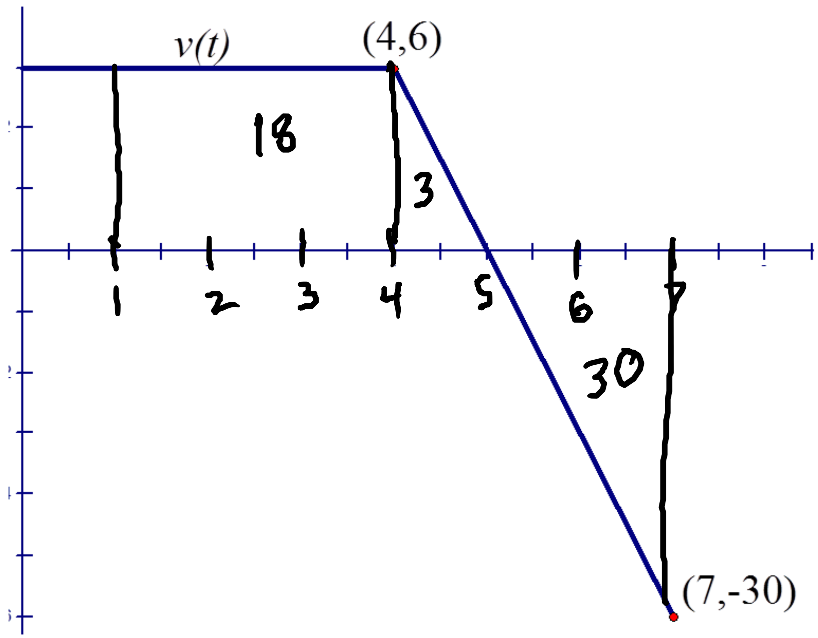
The integral of the rate of change gives the total change.

Ex. For a particle moving along the x-axis, you are given the graph of the velocity below. Assume $x(1) = 10$.

a) Find the total distance travelled on [1,7]. $\int_1^7 |v(t)|\,dt = 21+30 = 51$

b) Find $x(7)$. $= x(1) + \int_1^7 v(t)\,dt = 10 + 21 - 30 = 1$

c) When is the particle farthest to the left on [1,7]?



$x(1) = 10$

$x(5) = x(1) + \int_1^5 v(t)\,dt = 10+21 = 31$

$x(7) = 1$

$\boxed{t = 7}$

2013 AB#6

$\xrightarrow{\text{at } (1,0)}$ $\frac{dy}{dx} = e^0(3-6) = -3$

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

a) $y = f(1) + f'(1)(x-1)$

$y = 0 + (-3)(x-1)$

$f(1.2) \approx -3(1.2-1) = -.6$

b) $\frac{dy}{dx} = e^y(3x^2 - 6x)$

$\int e^{-y} dy = \int (3x^2 - 6x)\,dx$

$-e^{-y} = x^3 - 3x^2 + C$

$e^{-y} = -x^3 + 3x^2 + D$

$-y = \ln(-x^3 + 3x^2 + D)$

$y = -\ln(-x^3 + 3x^2 + D)$

$0 = -\ln(-1 + 3 + D)$

$e^0 = 2 + D$

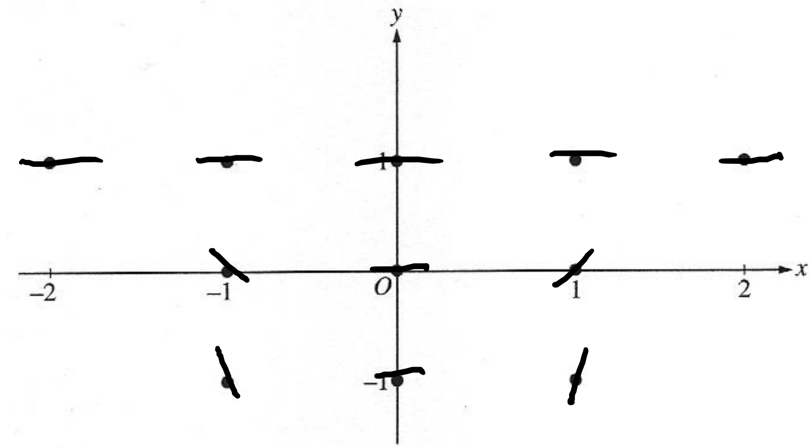
$D = -1$

$\boxed{y = -\ln(-x^3 + 3x^2 - 1)}$

Slope Fields

Ex. Consider $\frac{dy}{dx} = x(y-1)^2$

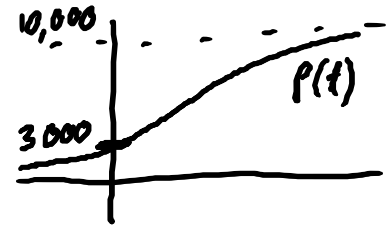
Sketch a slope field at the points indicated



<u>Ex.</u> A population is modeled by $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$ with $P(0) = 3{,}000$

$$2 - \frac{P}{5000} = 0$$

$$P = 10{,}000$$



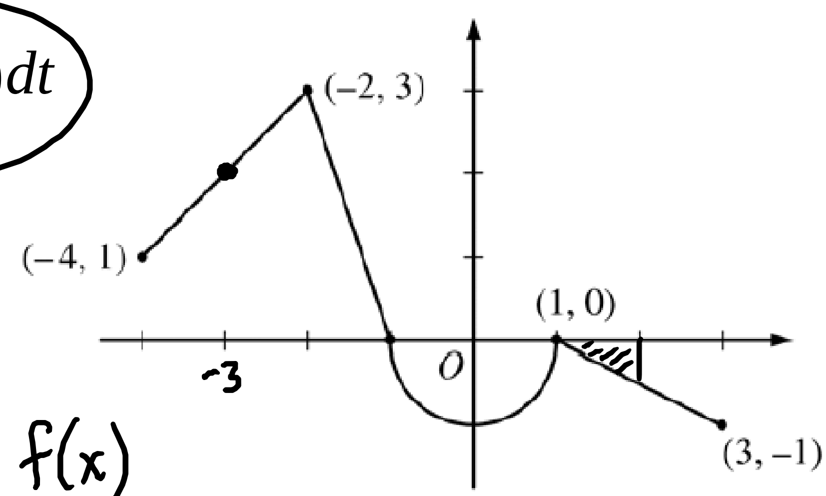
a. Evaluate $\lim_{t\to\infty} P(t) = 10{,}000$

b. What is the population when the population is growing most rapidly.

$$P = 5{,}000$$

Ex. Let f be the continuous function whose graph is shown.

Let g be the function $g(x) = \int_1^x f(t)\,dt$



Graph of f

$g'(x) = f(x)$

$g''(x) = f'(x)$

a) Find $g(2)$. $= \int_1^2 f(t)\,dt$

$= -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$

b) Find $g'(-3)$. $= f(-3)$

$= 2$

c) Find $g''(-3)$. $= f'(-3)$

$= 1$

SAMPLE B

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

a) $x'(2) = .271$

right because $x'(2) > 0$

slope $= \frac{y'(2)}{x'(2)} = 3.055$

b) $x(4) = x(2) + \int_2^4 x'(t)\,dt$

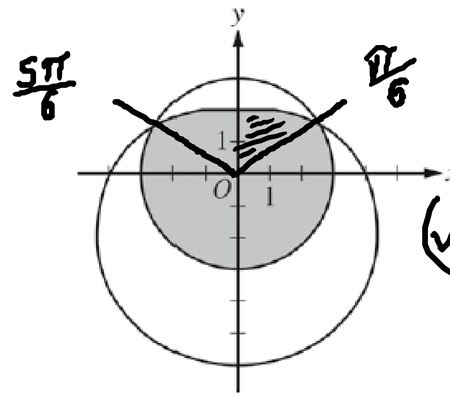
$= 1.253$

c) speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = .575$

accel. $= (x''(4), y''(4)) = (-.041, .989)$

d) $\int_2^4 \sqrt{(x')^2 + (y')^2}\,dt = .651$

2013 #2



c) $\left((4-2\sin t^2)\cos t^2,\ (4-2\sin t^2)\sin t^2\right)$

position

(veloc. at $t=1.5$) $= (-8.072, -1.673)$

The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

a) $A = 2\left[\frac{1}{2}\int_{\pi/6}^{\pi/2}(4-2\sin\theta)^2\,d\theta + \frac{1}{2}\int_{-\pi/2}^{\pi/6}(3)^2\,d\theta\right] = 24.709$

b) $x = r\cos\theta$

$x = (4-2\sin\theta)\cos\theta$

$x = (4-2\sin t^2)\cos t^2 = -1$

$t = 1.428$