If you want to tutor students at CHS:

https://tinyurl.com/CHSTutorMath

# L'Hopital's Rule

Ex. $$\lim_{x \to 1} \frac{\ln x \to 0}{x - 1 \to 0}$$

We can use a table of values, but there's an easier way.

Thm. L'Hopital's Rule

Consider $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)}$. If $f(x) \to 0$ and $g(x) \to 0$ as $x \to a$,

then

$$\boxed{\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}}$$

→Do not write "$= \frac{0}{0}$"

→You must evaluate top and bottom limits individually

→Also works when the limit is of the form $\frac{\infty}{\infty}$

<u>Ex.</u> $\lim\limits_{x\to1}\frac{\ln x}{x-1} \overset{L}{=} \lim\limits_{x\to1}\frac{1/x}{1} = \frac{1}{1} = \boxed{1}$

$$\boxed{\begin{gathered}\lim_{x\to1}\ln x = 0\\ \lim_{x\to1}(x-1) = 0\end{gathered}}$$

- This is not the quotient rule!
- Always plug in the number first to be sure that LHR applies.

Ex. $\lim_{x\to 2}\frac{x^4-16}{x-2} \overset{L}{=} \lim_{x\to 2}\frac{4x^3}{1} = 32$

$\lim_{x\to 2}(x^4-16)=0$

$\lim_{x\to 2}(x-2)=0$

Ex. $\lim_{x\to\infty} \frac{x^2}{e^x} \overset{L}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{L}{=} \lim_{x\to\infty} \frac{2}{e^x} = 0$

$$\boxed{\begin{array}{l} \lim_{x\to\infty} x^2 = \infty \\ \lim_{x\to\infty} e^x = \infty \end{array}} \qquad \boxed{\begin{array}{l} \lim_{x\to\infty} 2x = \infty \\ \lim_{x\to\infty} e^x = \infty \end{array}}$$

Ex. $\lim_{x \to 5} \frac{2x-3}{x+4} = \frac{7}{9}$

Ex. $\lim_{x \to 0} \frac{e^x}{x^2} = \frac{1}{+0} = \infty$

$\lim_{x \to 0} \frac{e^x}{x^3} = \frac{1}{\pm 0} = DNE$

**L'Hôpital (I have Calculus in the Heart)**

Once upon a time I had trouble with math,
But now they all think that I am smart,
There's nothing I can't do,
I have Calculus in the heart.
Once upon a time I was crying all night,
But now I do my math in the dark.
Nothing I can say,
I have Calculus in the heart.

L'Hôpital:
Of the top and of the bottom,
We don't need the algebra,
We're just go-ing to compare them,
L'Hôpital:
Cause we take the limit again,
If we find out that we cannot define
Our limit, we will have to go repeat one more time.
They almost always work, but if that's confusing,
An Oscillating Function.

Every now and then I get a little bit of trouble when I'm taking a limit:
And I know that we're making this strange,
Every now and then I get a zero for the numerator and the denominator.
Every now and then I get it, but if you're in the dark, some kind of indeterminate form.
Function may be leaving its mark!
L'Hôpital:
And then this doesn't work!
Every now and then I get a little bit terrified but then I think of all your advice.
But most of the time, well it does,
For most of the time our song works.
L'Hôpital:
Guillaume François Antoine Marquis de
L'Hôpital:
Guillaume François Antoine Marquis de L'Hôp!

Pract. $\lim_{x \to 0} \frac{\sin x}{x}$

Pract. $\lim_{x \to \infty} \frac{\ln x}{x^2}$

Pract. $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$

Pract. $\lim_{x \to 0} \frac{x^3}{\cos x}$