Warm up Problems

Let $f(x) = x^3 - 3x + 1$.

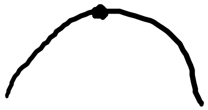
1) Find and classify all critical points.

2) Find all inflection points.

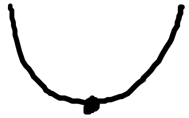
Graph of a Function, Part 2

Second Derivative Test

If p is a critical point of $f(x)$ and $f''(p) < 0$, then p is a local maximum.



If p is a critical point of $f(x)$ and $f''(p) > 0$, then p is a local minimum.



Ex. Find and classify all critical points of

$$f(x) = x^3 - 5x^2 + 3x - 1.$$

$f'(x) = 3x^2 - 10x + 3$

$= (3x-1)(x-3) = 0$

$x = \frac{1}{3}$ $x = 3$

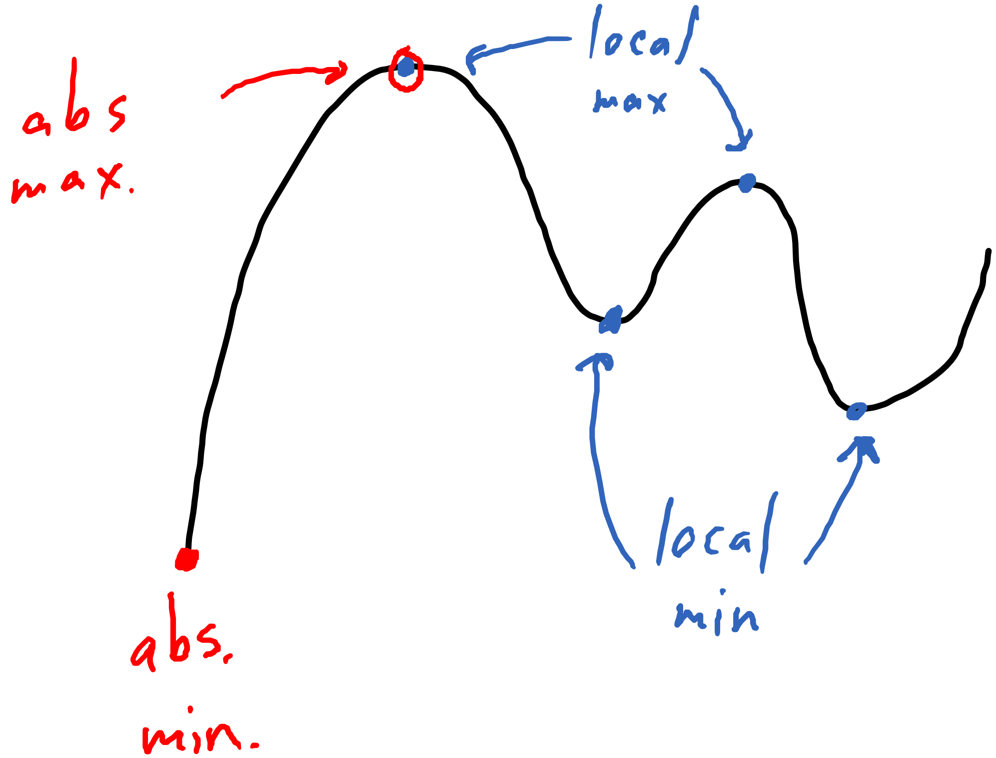
$f''(x) = 6x - 10$

$f''\left(\frac{1}{3}\right) = -8 \leftarrow$ local max

$f''(3) = 8 \leftarrow$ local min.

Def. The absolute maximum (global max) value of a function on an interval is the largest value that the function attains.

Def. The absolute minimum (global min) value of a function on an interval is the smallest value that the function attains.


abs max.
local max
local min
abs. min.

<u>Thm.</u> The absolute max. and min. will occur at one of the following:

- the point p where $f'(p) = 0$
- the point p where $f'(p)$ is undef.

} critical points

- an endpoint of the interval

Ex. Find the absolute max. and min. values of $f(x) = x^3 - 3x^2 + 1$ on $\left[-\frac{1}{2}, 4\right]$.

$f'(x) = 3x^2 - 6x$

$= 3x(x-2) = 0$

$x = 0 \quad x = 2$

$f(-\frac{1}{2}) = .125$

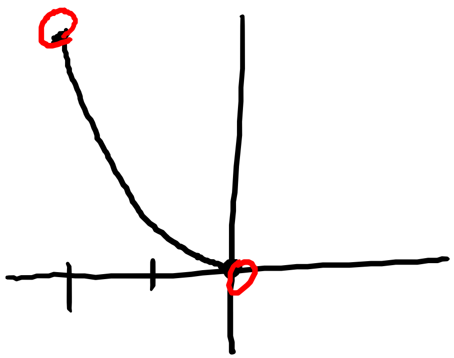
$f(0) = 1$

$f(2) = -3$

$f(4) = 17$

~~abs. max. value is 17~~ none
abs. min. value is -3

Ex. Find the *x*-coordinate of all local max./min. and absolute max./min. of $f(x) = x^2$ for $-2 \leq x \leq 0$ by graphing.



local max. → none

local min. → none

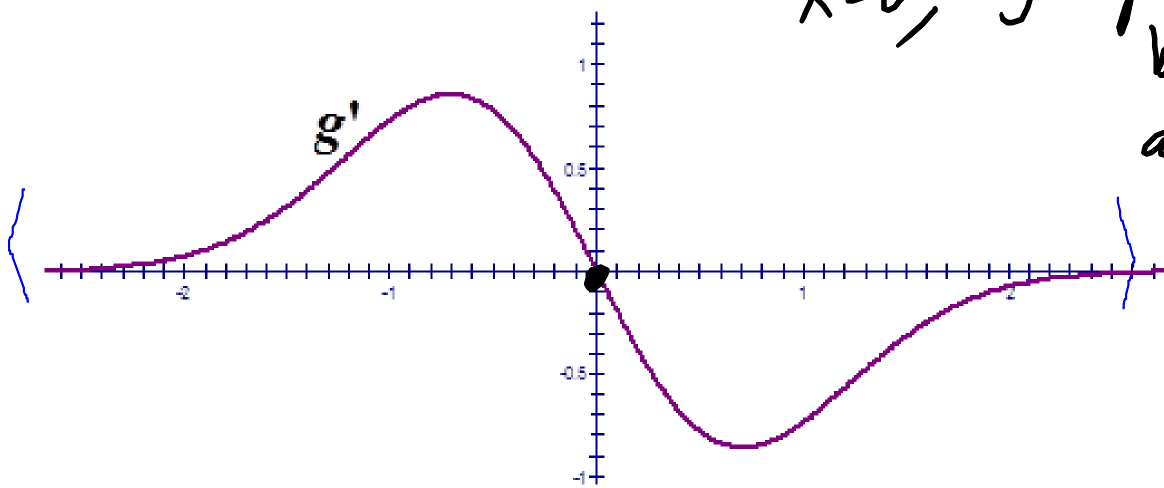
abs. max. → ~~x = −2~~ none

abs. min. → ~~x = 0~~ none

→What about open intervals?

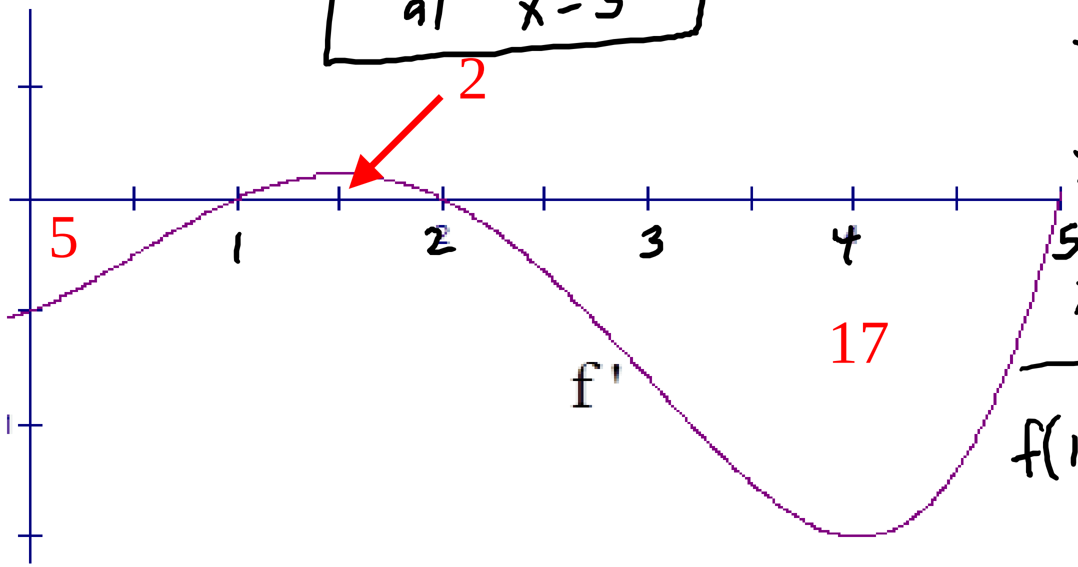
Ex. Find the x-coordinate of the absolute maximum of $g(x)$. Justify your answer.

$x=0$, g' pos. for all x before $x=0$ and neg. for all x after 0



Ex. Find the x-coordinate of the absolute minimum of $f(x)$ on [0,5]. Justify your answer.

abs. min. is at $x=5$



$x=0$: f' neg. after

$x=1$: $f(1)=4$

$x=2$: local max. (f' goes pos. to neg)

$x=5$: $f(5)=-11$

$f(1)=f(0)+\int_0^1 f'(x)\,dx$

$= 9+(-5)=4$

$f(5)=f(0)+\int_0^5 f'(x)\,dx$

$= 9+(-5+2-17)=-11$

$f(0)=9$

You must check ALL candidates.