Warm-up Problems

1. $\dfrac{(n+1)!}{n!} = \dfrac{\cancel{1 \cdot 2 \cdot 3 \cdot \ldots \cdot n} \cdot (n+1)}{\cancel{1 \cdot 2 \cdot 3 \cdot \ldots \cdot n}} = n+1$

2. $\dfrac{[2(n+1)]!}{(2n)!} = \dfrac{\cancel{1 \cdot 2 \cdot 3 \cdot \ldots \cdot (2n)}(2n+1)(2n+2)}{\cancel{1 \cdot 2 \cdot 3 \cdot \ldots \cdot (2n)}} = (2n+1)(2n+2)$

3. $3^{2n} = (3^2)^n = 9^n$

Ratio and Root Tests

<u>Thm.</u> Ratio Test

Let $\sum a_n$ be a series with nonzero terms.

i) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$, the series is abs. convergent.

ii) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$, the series is divergent.

iii) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, then the test fails.

Ex. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$

$$\lim_{n\to\infty}\left|\frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}}\right| = \lim_{n\to\infty}\left|\frac{\cancel{2^{n+1}}^{\,2}}{\cancel{(n+1)!}_{\,n+1}}\cdot\frac{\cancel{n!}^{\,1}}{\cancel{2^n}}\right| = \lim_{n\to\infty}\left|\frac{2}{n+1}\right| = 0 < 1$$

$\therefore \sum \frac{2^n}{n!}$ conv. by Ratio Test

<u>Ex.</u> $$\sum_{n=0}^{\infty} \frac{n^2 2^{n+1}}{(-3)^n}$$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^2\, 2^{n+2}}{(-3)^{n+1}}}{\frac{n^2\, 2^{n+1}}{(-3)^n}} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^2\, \overset{2}{\cancel{2^{n+2}}}}{\underset{-3}{\cancel{(-3)^{n+1}}}} \cdot \frac{\cancel{(-3)^n}}{n^2\, \cancel{2^{n+1}}} \right|$$

$$= \lim_{n\to\infty} \left| \underbrace{\frac{(n+1)^2}{n^2}}_{1} \cdot \frac{2}{-3} \right| = \frac{2}{3} < 1$$

$\therefore$ conv. by Ratio Test

<u>Thm.</u> Root Test

Let $\sum a_n$ be a series.

i) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$, the series is abs. conv.

ii) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$, the series is divergent.

iii) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, then the test fails.

Ex. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

$\sqrt[n]{e^{2n}} = \left(e^{2n}\right)^{1/n} = e^{2n \cdot \frac{1}{n}} = e^2$

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{e^{2n}}{n^n}\right|} = \lim_{n\to\infty} \frac{e^2}{n} = 0 < 1$$

$\therefore$ conv. by Root Test

Next class, you will take a quiz on the convergence tests.