- Green part is out of 100

Let’s talk about calculus shirts.

Series

A <u>series</u> is the sum of the terms of a sequence.

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

A partial sum, S_n, adds only the first n terms.

$$S_1 = a_1$$
$$S_2 = a_1 + a_2$$
$$S_3 = a_1 + a_2 + a_3$$
$$\vdots$$
$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

→ If the partial sums converges to a value, that value is the sum of the infinite series

Ex. Find $\sum_{n=1}^{\infty} \frac{1}{2^n}$ using partial sums.

$S_1 = \frac{1}{2^1} = \frac{1}{2}$

$S_2 = \frac{1}{2^1} + \frac{1}{2^2} = \frac{3}{4}$

$S_3 = \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} = \frac{7}{8}$

$S_4 = \frac{15}{16}$

$S_5 = \frac{31}{32}$

$S_\infty = 1$

Ex. Determine if $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$ converges using partial sums.

$S_1 = \frac{1}{1} - \frac{1}{2}$

$S_2 = \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \frac{1}{3}\right) = 1 - \frac{1}{3}$

$S_3 = \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \frac{1}{4}\right) = 1 - \frac{1}{4}$

$S_\infty = \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{6}}\right) + \ldots = 1$

→ This is called a <u>telescoping series</u>.

A geometric series is of the form $\sum_{n=0}^{\infty} a \cdot r^n$

Thm. Geometric Series Test

If $|r| \geq 1$, then the series diverges.

If $|r| < 1$, then the series converges to $\frac{\text{first term}}{1 - r}$

→ When using this test, you must state the value of r

Determine if the series converges or diverges, and justify your answer. If it converges, determine what it converges to.

Ex. $$\sum_{n=0}^{\infty} \frac{4}{3^n} = \sum_{n=0}^{\infty} 4\left(\frac{1}{3}\right)^n$$ $r = \frac{1}{3}$, conv. by Geom. Series Test

$$= \frac{4}{1-\frac{1}{3}} = \frac{4}{2/3} = 6$$

Ex. $$\sum_{n=0}^{\infty} \left(\frac{4}{3}\right)^n$$ $r = \frac{4}{3}$, div. by Geom. Series Test

Find each value.

Ex. $0.\overline{85}$ $= .85858585\ldots$

$= .85 + .0085 + .000085 + \ldots = \dfrac{.85}{1-.01} = \dfrac{.85}{.99} = \boxed{\dfrac{85}{99}}$

$\times .01$ $\times .01$

Ex. $18 - 12 + 8 - \frac{16}{3} + \frac{32}{9} - \cdots$ $= \dfrac{18}{1-\left(-\frac{2}{3}\right)} = \dfrac{18}{\frac{5}{3}}$

$\times -\frac{2}{3}$

$= \dfrac{54}{5}$

Determine if the series converges or diverges, and justify your answer.

Ex. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{3}{\pi}\right)^{n}$

$r = \frac{3}{\pi}$, conv. by Geom. Series Test

Ex. $\displaystyle\sum_{n=0}^{\infty} (\sin e^{10})^{n}$

$r = \sin e^{10}$, conv. by Geom. series Test

Ex. For what values of x does $\displaystyle\sum_{n=0}^{\infty}\left(\frac{3}{x-2}\right)^{n}$ converge?

$$\left|\frac{3}{x-2}\right| < 1$$

$$3 < |x-2|$$

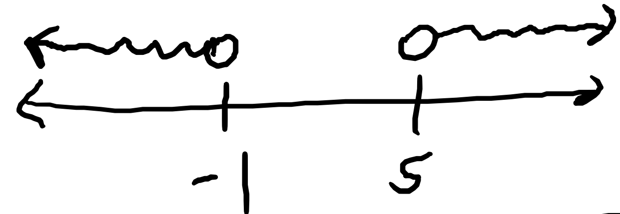
$$|x-2| = 3$$

$x-2=3$

$x=5$

$x-2=-3$

$x=-1$



$$\boxed{x<-1 \text{ or } x>5}$$

Thm. Test for Divergence

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=0}^{\infty} a_n$ diverges.

Ex. $\sum_{n=0}^{\infty} \frac{n}{n+1}$ $\quad \lim_{n\to\infty} \frac{n}{n+1} = 1 \quad$ div. by Test for Div.

Ex. $\sum_{n=1}^{\infty} 2^n$ $\quad \lim_{n\to\infty} 2^n = \infty \quad$ div. by Test for Div.

Ex. $\sum_{n=1}^{\infty} \frac{1}{n}$ $\quad \lim_{n\to\infty} \frac{1}{n} = 0 \quad$???