- Blue part is out of 50
- Green part is out of 50

→Total of 100 points possible

Differential Equations

Remember: A differential equation is an equation that involves a function and some of its derivatives.

Ex. Solve $y' = 6x^2 - 5$

$$y = 2x^3 - 5x + C$$

Ex. Verify that $x = 5$ is a solution to $3x - 2 = 13$.

$3(5) - 2 = 13$

$15 - 2 = 13$ ✓

Ex. Verify that $y = e^{2x}$ is a solution to $y'' - 3y' + 2y = 0$

$y' = e^{2x} \cdot 2$

$y'' = e^{2x} \cdot 4$

$4e^{2x} - 3(2e^{2x}) + 2(e^{2x}) = 0$

$4e^{2x} - 6e^{2x} + 2e^{2x} = 0$ ✓

A <u>separable</u> equation can be written

$$\frac{dy}{dx} = f(x)g(y)$$

→To solve, we treat $\frac{dy}{dx}$ as a fraction.

→Put the y's on the left with dy, put x's on the right with dx, and integrate each side.

Ex. Let $\frac{dP}{dt} = \frac{t^2}{2P^3}$, find the general solution.

$$\int P^3\,dP = \int \frac{1}{2} t^2\,dt$$

$$\frac{1}{4}P^4 = \frac{1}{6}t^3 + C$$

$$P^4 = \frac{2}{3}t^3 + D$$

$$P = \pm\sqrt[4]{\frac{2}{3}t^3 + D}$$

$$\frac{1}{4}P^4 = \frac{1}{6}t^3$$

$$P^4 = \frac{2}{3}t^3$$

$$P = \pm\sqrt[4]{\frac{2}{3}t^3} + C$$

Ex. Solve $\frac{dy}{dx} = \frac{y}{1+x}$

$$\int \frac{1}{y}\,dy = \int \frac{1}{1+x}\,dx$$

$$e^{\ln|y|} = e^{\ln|1+x| + C}$$

$$D = \pm e^{C}$$

$$|y| = e^{\ln|1+x|} \cdot e^{C}$$

$$y = D|1+x|$$

<u>Pract.</u> 1) $\frac{dR}{dt} = \frac{t^2}{R}$

$R\,dR = t^2\,dt$

$\frac{1}{2}R^2 = \frac{1}{3}t^3 + C$

$R^2 = \frac{2}{3}t^3 + D$

$R = \sqrt{\frac{2}{3}t^3 + D}$

2) $\frac{dy}{dx} = 12y^2$

$\frac{1}{y^2}\,dy = 12\,dx$

$-\frac{1}{y} = 12x + C$

$\frac{1}{y} = -12x + D$

$y = \frac{1}{D - 12x}$

Def. An initial value problem (IVP) is a differential equation and a value of the solution.

Ex. Solve the IVP $\frac{dP}{dt} = 3P$ if $P(0) = 5$.

$$\int \frac{1}{P}\,dP = \int 3\,dt$$

$$e^{\ln|P|} = e^{3t + C}$$

$$|P| = e^{3t} \cdot e^{C}$$

$$P = De^{3t}$$

$$5 = De^{3(0)}$$

$$D = 5$$

$$\boxed{P = 5e^{3t}}$$

Important: A solution must be a function that is differentiable over the largest interval containing the initial value and satisfying the original differential equation.

Ex. Find the solution to $\frac{dy}{dx} = \frac{x}{y}$ if $y(5) = -3$

$\int y\,dy = \int x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

$y^2 = x^2 + D$

$y = \pm\sqrt{x^2 + D}$

$-3 = \pm\sqrt{5^2 + D}$

$9 = 25 + D$

$D = -16$

$y = -\sqrt{x^2 - 16}$

~~$(-\infty, 4]$~~ $[4, \infty)$

$(4, \infty)$

Ex. Solve the IVP $\frac{dy}{dx} = \frac{y+1}{x}$, $y(1) = 2$, then sketch the solution.

$$\frac{dy}{dx} = \frac{y+1}{x}$$

$$\frac{1}{y+1}\,dy = \frac{1}{x}\,dx$$

$$\ln|y+1| = \ln|x| + C = e^{\ln|x|} \cdot e^{C}$$

$$e \qquad e$$

$$y+1 = D|x|$$

$$y = D|x| - 1 \qquad \cancel{(-\infty, 0)}\ (0, \infty)$$

$$2 = D|1| - 1$$

$$D = 3$$

$$y = 3|x| - 1$$