Other Applications

Thm. The average value of a function $f(x)$ over the interval $[a, b]$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Ex. Find the average value of $f(x) = \sin 5x$ on $[10,30]$.

$$\frac{1}{30-10}\int_{10}^{30} \sin 5x\,dx$$

Ex. The temperature, in $°C$, of a pond is a function W of time t. The table below shows the temperature at selected times. Approximate the average temperature over the time interval $0 \le t \le 15$ using right-hand sums with 5 subintervals.

t	$W(t)$
0	20
3	31
6	28
9	24
12	22
15	21

$$\frac{1}{15}\int_0^{15} w(t)\,dt$$

$$= \frac{1}{15}\left[3 \cdot W(3) + 3 \cdot W(6) + 3W(9) + 3W(12) + 3W(15)\right]$$

$$= 25.2\ °C$$

Def. The arc length of a curve on an interval $[a, b]$ is the length of the curve over the interval.

Thm. The arc length, s, of $f(x)$ on $[a, b]$ is given by

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

<u>Ex.</u> Find the length of $y = x^{3/2}$ on $[0,5]$.

$$y' = \frac{3}{2}x^{1/2}$$

$$S = \int_0^5 \sqrt{1 + \left(\frac{3}{2}x^{1/2}\right)^2}\,dx = \int_0^5 \sqrt{1 + \frac{9}{4}x}\,dx$$

$$u = 1 + \frac{9}{4}x$$

$$du = \frac{9}{4}dx$$

$$\frac{4}{9}du = dx$$

$$= \int_{*}^{**} u^{1/2} \cdot \frac{4}{9}du = \frac{4}{9}\cdot\frac{2}{3}u^{3/2}\Big|_{*}^{**}$$

$$= \frac{8}{27}\left(1 + \frac{9}{4}x\right)^{3/2}\Big|_0^5 = \frac{8}{27}\left(1 + \frac{45}{4}\right)^{3/2} - \frac{8}{27}(1)^{3/2}$$

<u>Ex.</u> Find the length of $f(x) = \int_1^x 2\sqrt{4t^2 + 2t}\,dt$ on [1,2].

$$f'(x) = 2\sqrt{4x^2+2x}$$

$$s = \int_1^2 \sqrt{1+\left[2\sqrt{4x^2+2x}\,\right]^2}\,dx = \int_1^2 \sqrt{1+4(4x^2+2x)}\,dx$$

$$= \int_1^2 \sqrt{16x^2+8x+1}\,dx = \int_1^2 \sqrt{(4x+1)^2}\,dx = \int_1^2 (4x+1)\,dx$$

$$= 2x^2+x\Big|_1^2 = (8+2)-(2+1) = \boxed{7}$$

Ex. Find the length of $y = \frac{x^3}{6} + \frac{1}{2x}$ on $[1,2]$.

$$= \frac{1}{6}x^3 + \frac{1}{2}x^{-1}$$

$$y' = \frac{1}{2}x^2 - \frac{1}{2}x^{-2}$$

$$S = \int_1^2 \sqrt{1 + \left(\frac{1}{2}x^2 - \frac{1}{2}x^{-2}\right)^2}\, dx$$