New seats today, you may sit where you wish.

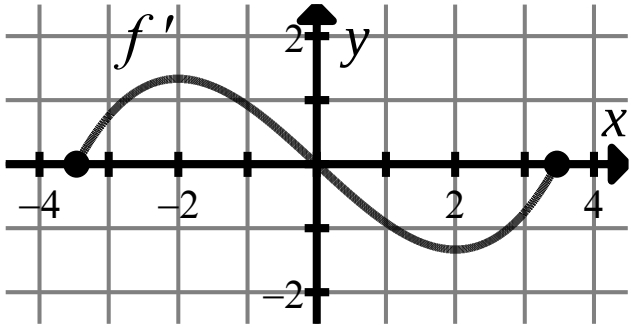
- Blue part is out of 37
- Green part is out of 65

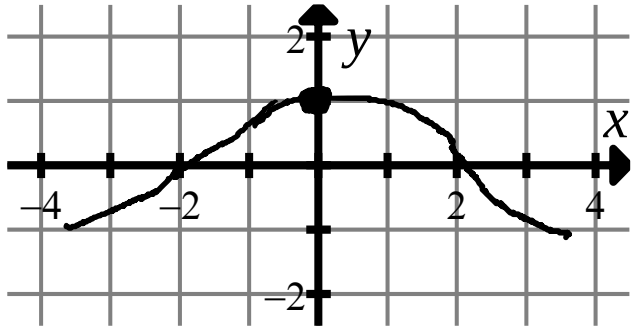
→Total of 102 points possible

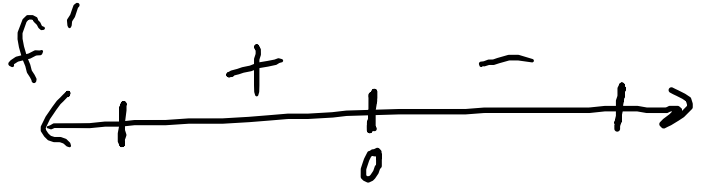
→Grade is out of 100

Antiderivatives

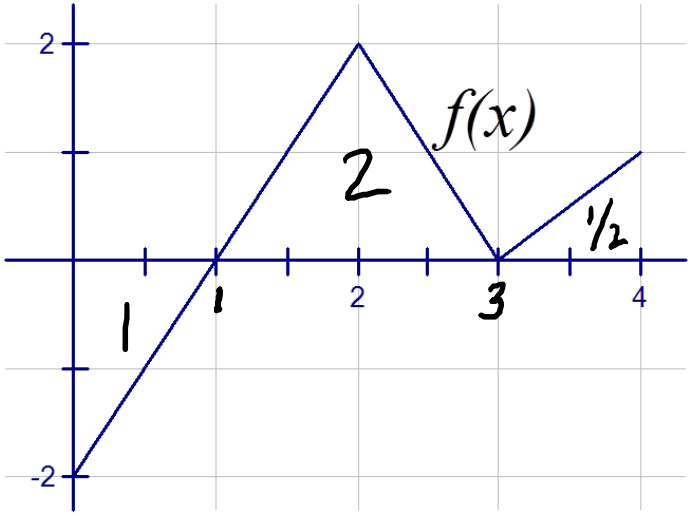
Ex. Use number lines for f' and f'' to sketch a graph of f with a starting point of (0,1).

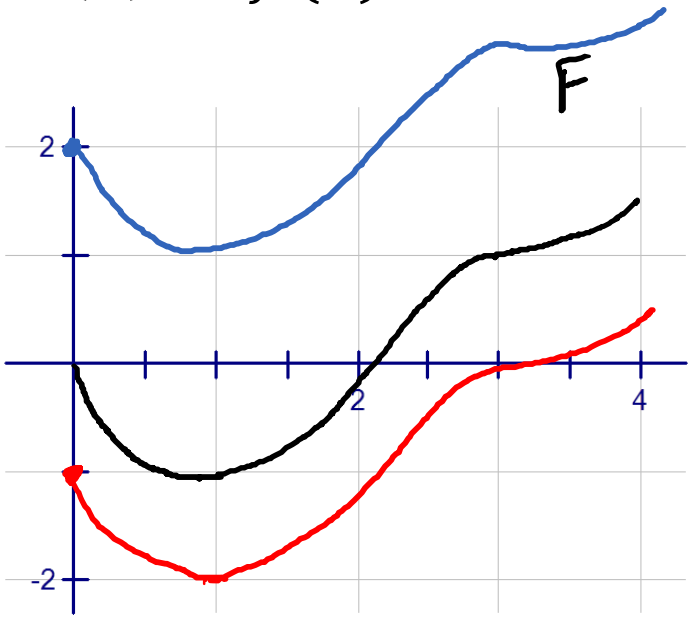






Ex. Let $F(x)$ be such that $F'(x) = f(x)$. Sketch a graph of $F(x)$.





$F(x)$ is called an antiderivative of $f(x)$.

→Notice that antiderivatives are not unique.

Computing Antiderivatives

Ex. Find an antiderivative of $3x^2$.

x^3

Ex. Find an antiderivative of x^5.

$\frac{1}{6}x^6$

The last answer was $\frac{1}{6}x^6$

→It could have been $\frac{1}{6}x^6 + 9$ or $\frac{1}{6}x^6 - 58$

To describe all possible answers, we write

$$\frac{1}{6}x^6 + c$$

→ This is called the general antiderivative.

Pract. Find the general antiderivative of $x^2 - 4$.

$\frac{1}{3}x^3 - 4x + C$

Def. The indefinite integral of $f(x)$, written $\int f(x)dx$, is the general antiderivative of $f(x)$.

Ex. $\int x^5 dx = \frac{1}{6}x^6 + c$

"find the integral" requires "+ c"
"find an antiderivative" doesn't need "+ c"

$$\int f(x)dx \quad \underline{\text{vs.}} \quad \int_a^b f(t)dt$$

Indefinite integral	Definite integral
Has no endpoints	Has endpoints
Is a function	Is a number
General antiderivative	Area under the curve

Integral Rules

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

$$\int cf(x)dx = c\int f(x)dx$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c \quad \text{for } n \neq -1$$

Ex. $\int (4x^2 - x^3)dx = \frac{4}{3}x^3 - \frac{1}{4}x^4 + C$

Ex. $\int \left(5x^3 - \frac{2}{x^2} + 10\right)dx = \int (5x^3 - 2x^{-2} + 10)dx$

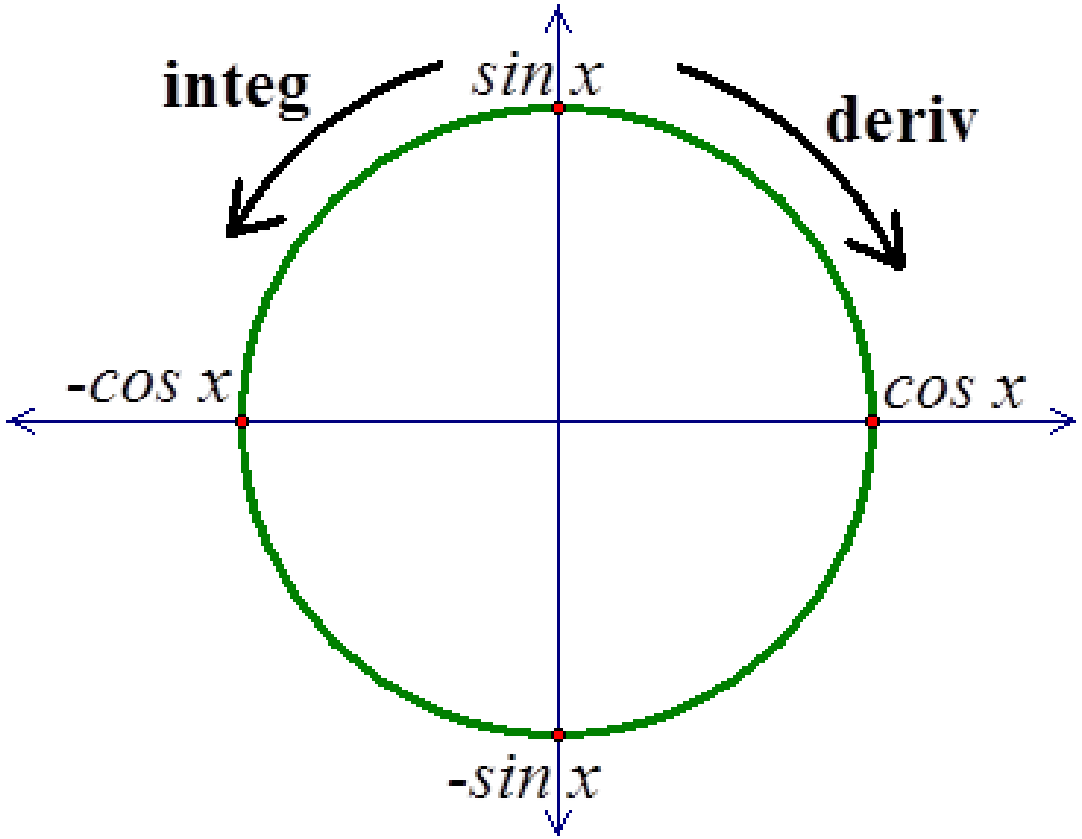
$= \frac{5}{4}x^4 + 2x^{-1} + 10x + C$

Pract. Find the following in groups:

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int e^x dx = e^x + C \qquad \int \sec^2 x\,dx = \tan x + C$$

$$\int \frac{1}{x} dx = \ln|x| + C \qquad \int 1\,dx = x + C$$


integ
deriv
sin x
-cos x
cos x
-sin x

Ex. If $f'(x) = \frac{1}{x}$ and $f(1) = 3$, find $f(x)$.

$f(x) = \ln|x| + C$

$f(1) = \ln 1 + C = 3$

$C = 3$

$\boxed{f(x) = \ln|x| + 3}$