

# Warm up Problems

$$1. \int_1^x (2t - 3) dt = t^2 - 3t \Big|_1^x = (x^2 - 3x) - (1^2 - 3) = x^2 - 3x + 2$$

$$2. \frac{d}{dx} \left[ \int_1^x (2t - 3) dt \right] = \frac{d}{dx} (x^2 - 3x + 2) = 2x - 3$$

$$3. \int_{10}^x f'(t) dt = f(t) \Big|_{10}^x = f(x) - f(10)$$

$$4. \frac{d}{dx} \int_{10}^x f'(t) dt = \frac{d}{dx} [f(x) - f(10)] = f'(x)$$

## Second FTOC

$F(x) = \int_a^x f(t) dt$  is called an integral function.

$$F'(x) = f(x)$$

Ex. Let  $F(x) = \int_3^x te^{-t} dt \longrightarrow F'(x) = \frac{d}{dx} \int_3^x te^{-t} dt$

a) Find  $F(5) = \int_3^5 te^{-t} dt = .159$   $\left. \vphantom{\int_3^5} \right\} = xe^{-x}$

b) Find  $F'(5) = 5e^{-5} = .034$

c) Find a value of  $x$  where  $F(x) = 0$   $x=3$

d) Find  $F''(5) = -.027$

$$\begin{aligned} \underline{\text{Ex.}} \quad \frac{d}{dx} \int_x^2 \left( \frac{1}{t} - \sin t \right) dt &= - \frac{d}{dx} \int_2^x \left( \frac{1}{t} - \sin t \right) dt \\ &= - \left( \frac{1}{x} - \sin x \right) \end{aligned}$$

$$\underline{\text{Ex.}} \quad \frac{d}{dt} \int_3^{5x} \cos^2 t \, dt = \cos^2(5x) \cdot 5$$

$$\underline{\text{Ex.}} \quad \frac{d}{d~~x~~} \int_{5x}^{x^2} e^t \sin t \, dt = e^{x^2} \sin(x^2) \cdot 2x - e^{5x} \sin 5x \cdot 5$$

Ex. Let  $f$  be the continuous function whose graph is shown.

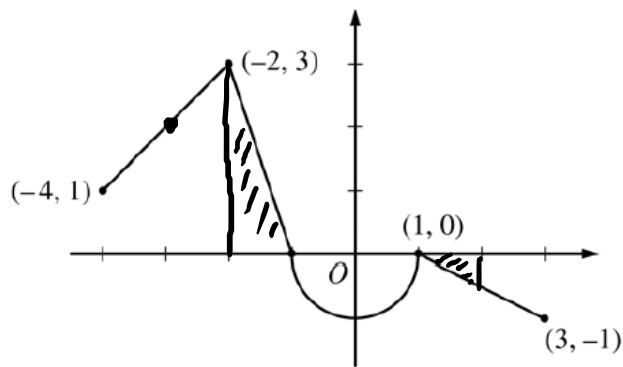
Let  $g$  be the function  $g(x) = \int_1^x f(t) dt$

a) Find  $g(2)$  and  $g(-2)$ .

$$g(2) = \int_1^2 f(t) dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$$

$$g(-2) = \int_1^{-2} f(t) dt = -\int_{-2}^1 f(t) dt = -\left[\frac{1}{2} \cdot 1 \cdot 3 - \frac{1}{2}\pi \cdot 1^2\right] = -\frac{3}{2} + \frac{\pi}{2}$$

b) Find  $g'(-3)$  and  $g''(-3)$ .  $g'(x) = f(x)$        $g''(x) = f'(x)$   
 $g'(-3) = f(-3) = 2$        $g''(-3) = f'(-3) = 1$



Graph of  $f$

$$g(x) = \int_1^x f(t) dt$$

$$g' = f$$

c) Find the  $x$ -coord. of each point where  $g$  has a horiz. tangent line. Classify each point and justify your answer.

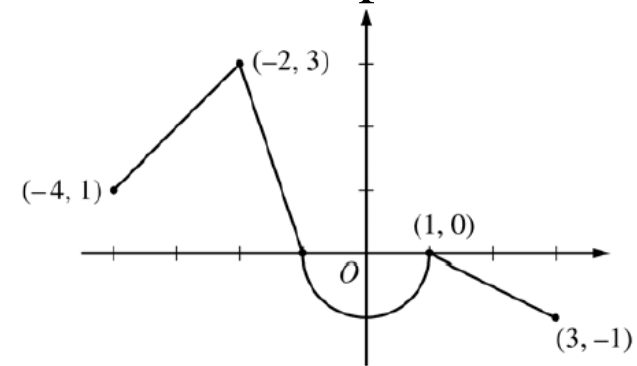
$x=1 \rightarrow$  neither,  $f$  doesn't cross  $x$ -axis

$x=-1 \rightarrow$  local max.,  $f$  goes pos. to neg.

d) Find the  $x$ -coord. of each point where  $g$  has an inflection pt. and justify your answer.

$x=0, x=-2, x=1$

slope of  $f$  changes signs



Graph of  $f$

## Unit 6 Progress Check: MCQ Part A

- Do #1-4, 8-12