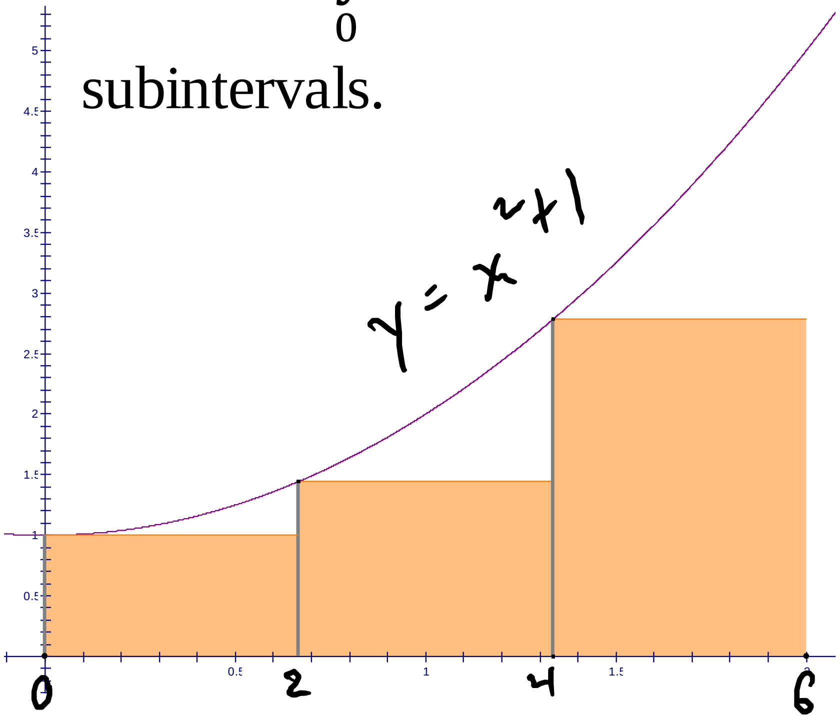
Riemann Sums

Ex. Approx. $\int_0^6 (x^2+1)\,dx$ using a left-hand Riemann sum with 3 subintervals.



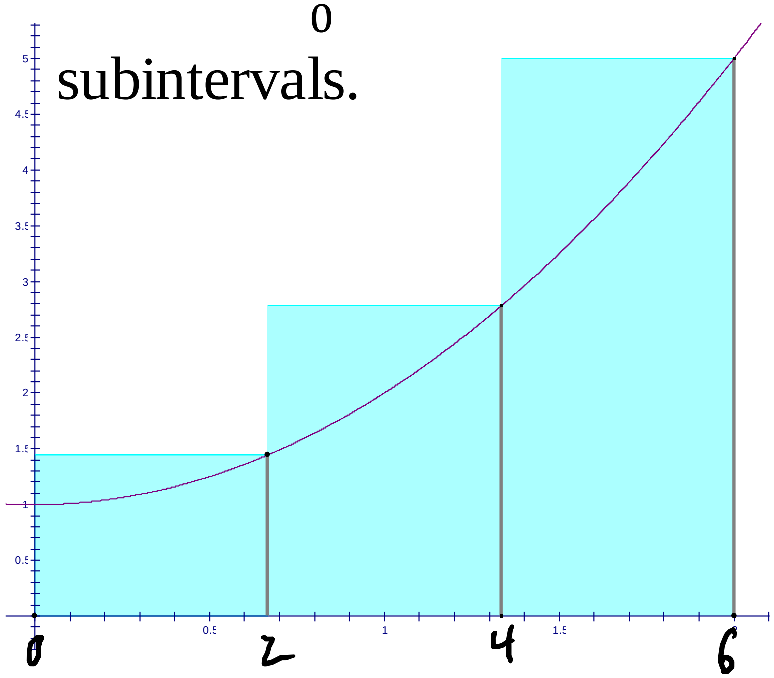
$2 \cdot f(0) + 2 \cdot f(2) + 2 \cdot f(4)$

$2 \cdot 1 + 2 \cdot 5 + 2 \cdot 17$

$2 + 10 + 34$

$\boxed{46}$

Ex. Approx. $\int_0^6 (x^2+1)\,dx$ using a right-hand Riemann sum with 3 subintervals.



$2\cdot f(2) + 2\cdot f(4) + 2\cdot f(6)$

$2\cdot 5 + 2\cdot 17 + 2\cdot 37$

$10 + 34 + 74$

118

Ex. For each of the previous examples, did we get an overestimate or an underestimate of the true value? Why?

Ex. The table below gives selected values of $f(x)$. Use these values and a left-hand Riemann sum to approximate the area under the function on the interval $0 \le x \le 12$.

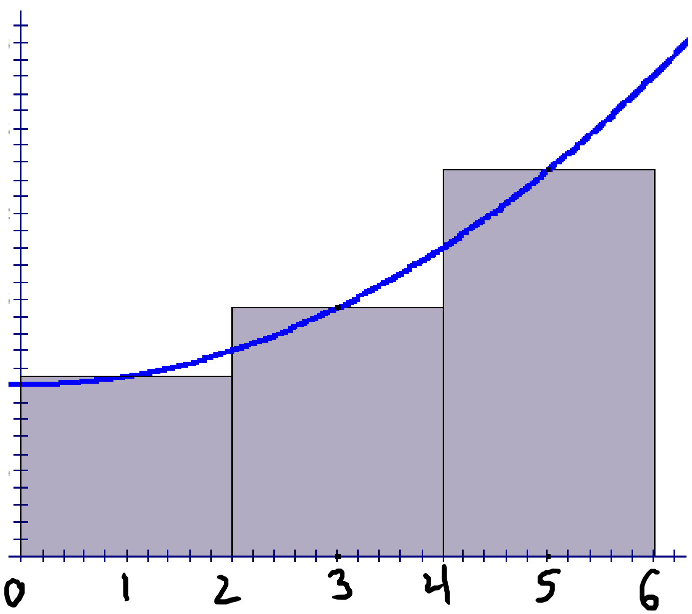
x	0	2	3	6	8	9	12
$f(x)$	0	.25	.48	.68	.84	.95	1

$$2 \cdot f(0) + 1 \cdot f(2) + 3 \cdot f(3) \cdot 2 \cdot f(6) + 1 \cdot f(8) + 3 \cdot f(9)$$

$$\boxed{6.74}$$

We can get a better approximation by using the midpoint:

Ex. Approx. $\int_0^6 (x^2+1)dx$ using a midpoint Riemann sum with 3 subintervals.



$2f(1) + 2\cdot f(3) + 2\cdot f(5)$

$2\cdot 2 + 2\cdot 10 + 2\cdot 26$

$4 + 20 + 52$

76

We could also use trapezoids:

$$A = \tfrac{1}{2}(h_1 + h_2)b$$

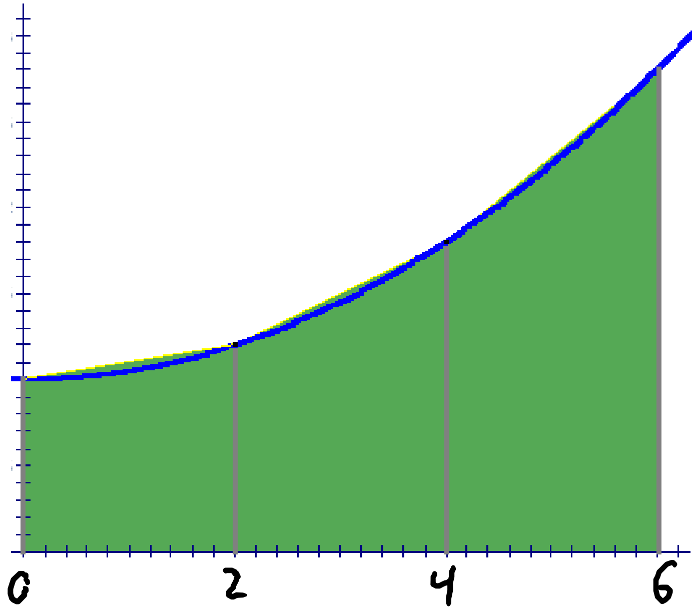
Ex. Approx. $\int_0^6 (x^2+1)\,dx$ using a trapezoidal Riemann sum with 3 subintervals.

$\frac{1}{2}\left[f(0)+f(2)\right]\cdot 2 + \frac{1}{2}\left[f(2)+f(4)\right]\cdot 2$
$+ \frac{1}{2}\left[f(4)+f(6)\right]\cdot 2$

$(1+5) + (5+17) + (17+37)$

$6 + 22 + 54$

$\boxed{82}$



Ex. For each of the previous examples, did we get an overestimate or an underestimate of the true value? Why?

left → under
right → over
f inc.

mid. → under
trap. → over
f ccu

Ex. Given values in the table, approx. the area under $f(x)$ on [0,8] using midpoint and trapezoidal Riemann sum with 4 subintervals of equal width.

x	0	1	2	3	4	5	6	7	8
$f(x)$	-1	0	3	4	7	9	14	16	20

$$2[f(1)+f(3)+f(5)+f(7)]$$

$$\boxed{58}$$