Warm up Problems

Assume for a continuous and differentiable function, $f(-1) = -2$ and $f(3) = 6$. Determine if the following statement is guaranteed by one of our theorems, then state the theorem that was used:

1) $f(c) = 1$ for some c on the interval $[-1,3]$. T, IVT
2) $f'(c) = 0$ for some c on the interval $[-1,3]$. F
3) $-2 \leq f(c) \leq 6$ for all c on the interval $[-1,3]$. F
4) $f'(c) = 2$ for some c on the interval $[-1,3]$. T, MVT

Review of Chapter 4

Strategy for Related Rates
"PGWEDA"

P) Draw a picture → Change = variable; Fixed = number

G) Ident... given i... ...luding rates

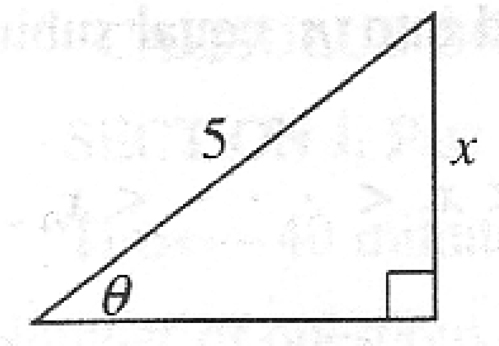
"Changes at a rate of …"

Involves time

E) ... ...es → Use the picture

D) Take the derivative with respect to time

A) Plug in values to get your answer



Ex. In the triangle shown, θ increases at a constant rate of 3 radians per minute. At what rate is x increasing when x equals 3 units?

$$\frac{d\theta}{dt} = 3$$

$$x = 3$$

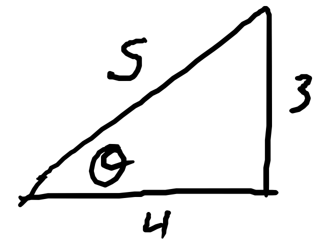
$$\frac{dx}{dt} = ?$$

$$\sin\theta = \frac{x}{5}$$

$$\cos\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$$

$$\left(\frac{4}{5}\right)3 = \frac{1}{5}\frac{dx}{dt}$$

$$12 = \frac{dx}{dt}$$



Ex. $\displaystyle\lim_{x\to 0}\frac{\cos x - 1}{x} \overset{L}{=} \lim_{x\to 0}\frac{-\sin x}{1} = \boxed{0}$

$\displaystyle\lim_{x\to 0}(\cos x - 1) = 0$

$\displaystyle\lim_{x\to 0} x = 0$

Ex. $\displaystyle\lim_{x\to\infty}\frac{e^x}{x^2} \overset{L}{=} \lim_{x\to\infty}\frac{e^x}{2x} \overset{L}{=} \lim_{x\to\infty}\frac{e^x}{2} = \boxed{\infty}$

$\displaystyle\lim_{x\to\infty} e^x = \infty$

$\displaystyle\lim_{x\to\infty} x^2 = \infty$

$\displaystyle\lim_{x\to\infty} e^x = \infty$

$\displaystyle\lim_{x\to\infty} 2x = \infty$

Ex. $\displaystyle\lim_{x\to 0}\frac{e^x}{x^2} = \infty$

$\displaystyle\lim_{x\to 0} e^x = 1$

$\displaystyle\lim_{x\to 0} x^2 = +0$

$\displaystyle\lim_{x\to 0}\frac{e^x}{x^3} = \text{DNE}$

$\displaystyle\lim_{x\to 0} x^3 = \pm 0$

Ex. Let $f(x)=\frac{x-4}{x^2}$ for $x\neq 0$. Find and classify all critical points. Find all inflection points. Find the global max/min values on [1,100].

$f(x)=\frac{x}{x^2}-\frac{4}{x^2}$

$=x^{-1}-4x^{-2}$

$f'(x)=-x^{-2}+8x^{-3}$

$=-\frac{1}{x^2}+\frac{8}{x^3}$

$=\frac{-x+8}{x^3}=0$

$-x+8=0$

crit. pt. → $\boxed{x=8}$

f': − (0) + (8) −

← local max.

$f''(x)=2x^{-3}-24x^{-4}$

$=\frac{2}{x^3}-\frac{24}{x^4}$

$=\frac{2x-24}{x^4}=0$

$2x-24=0$

$x=12$

f'': − (0) − (12) +

← inf. pt.

$f(1)=\frac{-3}{1}=-3$

$f(8)=\frac{4}{8^2}=\frac{1}{16}=.0625$

$f(100)=\frac{96}{100^2}=.0096$

abs. min. value is -3
abs. max. value is $.0625$

Strategy for Optimization

1) Draw a picture, if appropriate

2) Writ… giv… ncluding an

Find the largest, smallest, closest

Maximize, minimize

5) Ta… the derivative

6) Set equal to zero and solve

Calculators Allowed

1. B 2. B 3. B 4. A 5. B

6. C 7. 375,000 $ft.^2$ 8. .321 rad/sec.

9. -5.890 cm^3/hr. 10. -1.014 m/s

No Calculators

1. D 2. C 3. E 4. A

5. a. (1,3) b. -3

6. a. x = -2 because $f\,'$ goes from pos. to neg.

b. x = 4 because $f\,'$ goes from neg. to pos.

c. -1 < x < 1 and 3 < x < 5 because f ○ is incr.

7. a. horiz. tan at $x = \sqrt{2}$ and $x = -\sqrt{2}$, both local min. because $f\,'$ goes from neg. to pos.

b. concave up for all $x \neq 0$ because $f\,'' > 0$

c. below, because graph is concave up

8. f twice-diff. → f continuous → IVT applies

$f(2) < 0 < f(4)$ → $f(c) = 0$ on interval

9. a. $x = 1$ and $x = 3$ because slope of f' changes signs

b. $x = 4$ because $f(0) = 9$, $f(4) = 2$, $f(5) = 6$, and $x = 1$ is not a max or min.

c. $y - 30 = 16(x - 5)$