Implicit Differentiation

<u>Explicit Functions</u> → $y = f(x)$

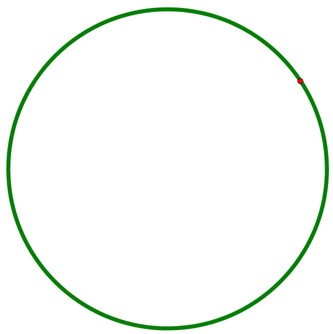
$y = \sin x + e^x$

$y = x^5 + 3x^2 + 2$

Find the slope of the graph at the point

<u>Implicit Functions</u> → implies $y = f(x)$

$x^2 + y^2 = 9$



Find the slope of the graph at the point

To differentiate an implicit function, we differentiate term-by-term:

- Take the derivative of x-function as usual.
- The derivative of y-function gets multiplied by y'.
- If x's and y's are in the same term, use product rule.

After differentiating, solve for y'.

Ex. If $\ln y + \underset{\sim}{x^2 y^4} + e^x = 5$, find $\dfrac{dy}{dx}$.

$$\frac{1}{y}y' + \underset{\sim}{x^2 \cdot 4y^3 y' + y^4 \cdot 2x} + e^x = 0$$

$$\frac{1}{y}y' + 4x^2y^3y' = -2xy^4 - e^x$$

$$y'\left(\frac{1}{y} + 4x^2y^3\right) = -2xy^4 - e^x$$

$$y' = \frac{-2xy^4 - e^x}{\frac{1}{y} + 4x^2y^3}$$

Ex. Find the slope of the line tangent to $y = x + \cos(xy)$ at the point where $x = 0$.

$y' = 1 - \sin(xy)(xy' + y \cdot 1)$

$y' = 1 - \sin(0 \cdot 1)(0y' + 1)$

$y' = 1$

$y = 0 + \cos(0 \cdot y)$

$y = 1$

$\underline{\text{Ex.}}$ If $\ln y + x^2y^4 + e^x = 5$, find $\dfrac{d^2y}{dx^2}$. $\rightsquigarrow y''$

$$\frac{dy}{dx} = \frac{-2xy^4 - e^x}{\frac{1}{y} + 4x^2y^3}$$

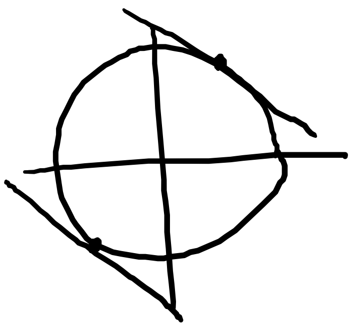
$$\frac{d^2y}{dx^2} = \frac{\left(\frac{1}{y} + 4x^2y^3\right)\left(-2x \cdot 4y^3 y' + y^4(-2) - e^x\right) - \left(-2xy^4 - e^x\right)\left(-y^{-2}y' + 4x^2 \cdot 3y^2 y' + y^3 \cdot 8x\right)}{\left(\frac{1}{y} + 4x^2y^3\right)}$$

Ex. Find the coordinates of any point on $x^2 + y^2 = 16$ where the tangent line has the slope of -1.

$$2x + 2yy' = 0$$

$$y' = -\frac{x}{y} = -1$$

$$y = x$$



$$x^2 + y^2 = 16$$

$$x^2 + x^2 = 16$$

$$2x^2 = 16$$

$$x^2 = 8$$

$$x = \pm\sqrt{8}$$

$$(\sqrt{8}, \sqrt{8}) \quad (-\sqrt{8}, -\sqrt{8})$$

Ex. Let $\cancel{f(x)}$ y $= x^3 + x$. If $g(x) = f^{-1}(x)$, and $f(2)=10$ find $g'(10)$.

$\boxed{g(x)}$

$g(10) = 2$

$x = y^3 + y$

$1 = 3y^2 y' + 1 \cdot y'$

$1 = 3(2)^2 y' + y'$

$1 = 13y'$

$y' = \frac{1}{13}$