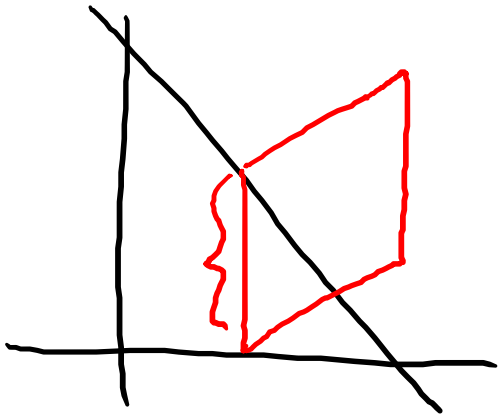
Volume

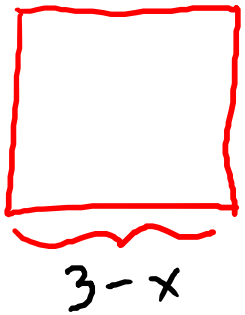
$$V = \int_a^b A(x)dx$$

Find the area of a random cross section, then integrate it.

Ex. Find the volume of the solid with base the region bounded by $y = 3 - x$, $x = 0$, and $y = 0$ whose cross-sections are squares perpendicular to the x-axis.

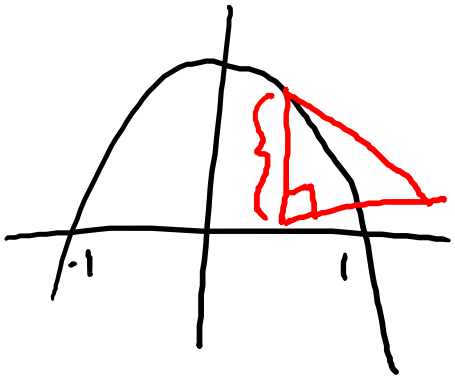


$$V = \int_0^3 (3-x)^2 \, dx$$

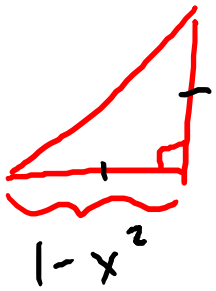


$$A = (3-x)^2$$

Ex. Base is the region bounded by $y = 1 - x^2$ and the x-axis; cross-sections are isosceles right triangles with leg perpendicular to the x-axis.



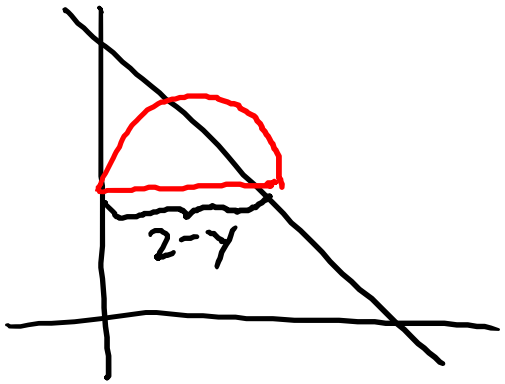
$$V = \int_{-1}^{1} \frac{1}{2}(1-x^2)^2 \, dx$$



$$A = \frac{1}{2}(1-x^2)^2$$

Ex. Base is the region bounded by $x + y = 2$ ~~and the~~ coordinate axes; cross-sections are semicircles with diameters perpendicular to the y-axis.

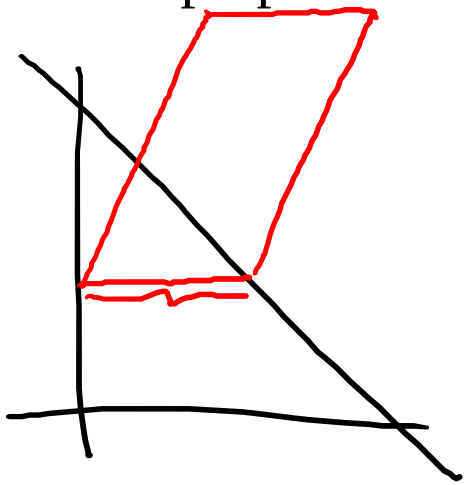
$x = 2 - y$



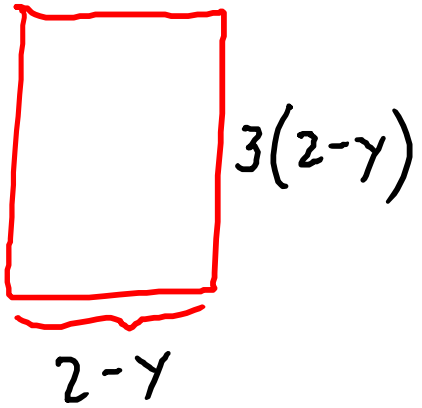
$$V = \int_0^2 \frac{\pi}{8}(2-y)^2\,dy$$

$$A = \frac{\pi}{2}\left(\frac{2-y}{2}\right)^2 = \frac{\pi}{8}(2-y)^2$$

Ex. Base is the region bounded by $x + y = 2$ and the coordinate axes; cross-sections are rectangles with heights that are 3 times the lengths that are perpendicular to the y-axis.



$$V = \int_0^2 3(2-y)^2\,dy$$



$$A = 3(2-y)^2$$