t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f, where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

   (a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t)\, dt$ in the context of the problem. Use a right Riemann sum with the three subintervals [60, 90], [90, 120], and [120, 135] to approximate the value of $\int_{60}^{135} f(t)\, dt$.

   (b) Must there exist a value of c, for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

   (c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right)\cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \le t \le 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \le t \le 150$.

   Show the setup for your calculations.

   (d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

2. Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t}\sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.

   (a) Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.

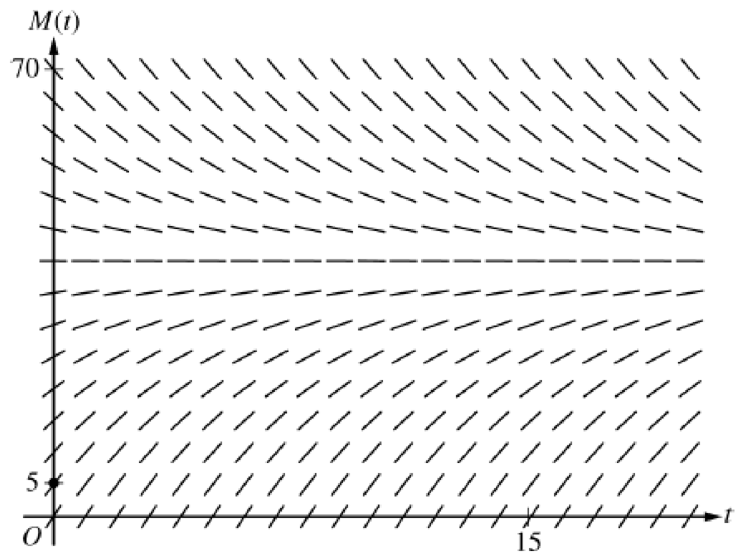
   (b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.

   (c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.

   (d) Find the total distance Stephen swims over the time interval $0 \le t \le 90$ seconds. Show the setup for your calculations.

3. A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

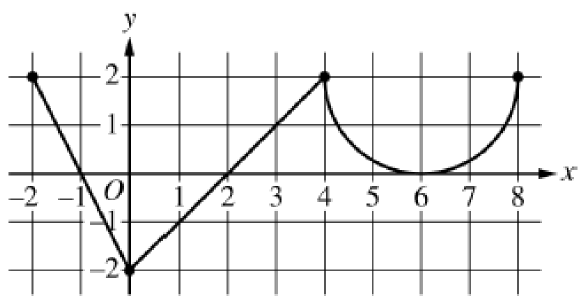
(a) A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.



(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.



Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

   (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

   (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

   (c) Find the value of $\lim_{x \to 2} \dfrac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

   (d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

   (a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

   (b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

   (c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\,dt$. Find $m(2)$. Show the work that leads to your answer.

   (d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

6. Consider the curve given by the equation $6xy = 2 + y^3$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{2y}{y^2 - 2x}$.

(b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.

(c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.

(d) A particle is moving along the curve. At the instant when the particle is at the point $\left(\dfrac{1}{2}, -2\right)$, its horizontal position is increasing at a rate of $\dfrac{dx}{dt} = \dfrac{2}{3}$ unit per second. What is the value of $\dfrac{dy}{dt}$, the rate of change of the particle's vertical position, at that instant?

<u>2023 AB Free Response Answers</u>

1)

a) Amount of gas, in gallons, pumped from $t = 60$ to $t = 135$; 8.25

b) $\frac{f(120)-f(60)}{120-60} = 0$; f is diff. and therefore cont., so c exists by MVT

c) 0.096

d) -0.005; rate at which gas flows into the tank is decreasing at a rate of 0.005 gal/sec/sec

2)

a) $t = 56$ because v changes signs

b) -0.036 m/sec/sec; speeding up because $v(60)$ and $a(60)$ are the same sign

c) 23.384

d) 62.164

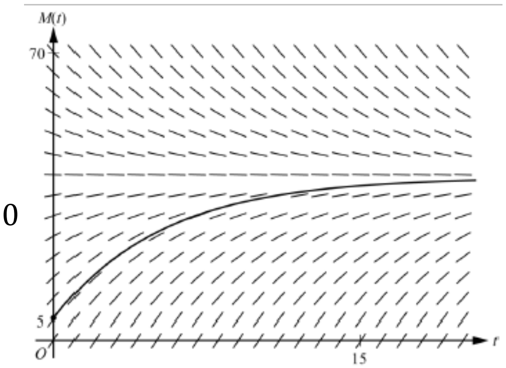
3)

a) →→→

b) 22.5

c) $\frac{d^2M}{dt^2} = -\frac{1}{16}(40 - M)$; overestimate because $\frac{d^2M}{dt^2} < 0$

d) $M = 40 - 35e^{-t/4}$



4)

a) Neither, f' doesn't change signs at $x = 6$

b) $(-2,0)$ and $(4,6)$ because f' is decreasing

c) 3

d) 1

5)

a) 12

b) Concave down, $k''(4) < 0$

c) 37

d) Increasing, $m'(2) > 0$

6)

a)

b) No points where tangent line is horizontal

c) $\left(\frac{1}{2}, 1\right)$

d) $-\frac{8}{9}$